Modified Fuzzy Data Envelopment Analysis Models

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Abstract
This paper examines the use of data envelopment analysis (DEA) in the conduct of efficiency measurement involving fuzzy (interval) input-output values. Data envelopment analysis is a linear programming method for comparing the relative productivity (or efficiency) of multiple service units. Standard DEA models assume crisp data for both the input and output values. In practice however, input and output values may be uncertain, vague, imprecise or incomplete. New pairs of fuzzy DEA (FDEA) models are presented which differ from existing fuzzy DEA models handling uncertain data. In this approach, upper bound interval data are used exclusively to obtain the upper frontier values while lower bound interval data are used exclusively to obtain the lower frontier values. The outcome, when compared with the outcome of existing approach, based on the same set of data, shows a swap in the upper and lower frontier values with exactly the same number of efficient decision making units (DMUs). This new approach therefore clears the ambiguity occasioned by the mixture of upper and lower bound values in the determination of the upper and lower frontier efficiency scores respectively. The modified FDEA models make application and interpretation of results easy. The most efficient units, for each of the models, have efficiency score of 1 with equivalent ranking score of 1. These efficient units also serve as reference sets to the inefficient units. The inefficient units have efficiency scores less than 1 for all the models. The most inefficient unit is S13 for all the models and it has the least efficiency score in each case and a ranking score of 25.

Keywords: Fuzzy, Data envelopment analysis, Modified, Models.

Introduction

This paper extends the technique of fuzzy (interval) data envelopment analysis (FDEA). In particular, the paper seeks to present modified fuzzy Charnes, Cooper and Rhodes (FCCR) and fuzzy Banker, Charnes and Cooper (FBCC) models for use with interval fuzzy numbers. This paper also compares the conventional DEA, the FDEA presented by Zeidan *et al*. (2016) and Demir (2014). For ease of comparison, the data set for 25 high schools in the 2012-2013 education year in Demir (2014) is used.

Data envelopment analysis (DEA) was first presented by Charnes, Cooper and Rhodes (1978) leveraging on the 1957 seminal paper of Farrell whose main purpose was the estimation of technical efficiency and efficiency frontiers. DEA has become one of the most widely used techniques for measuring the efficiency of decision making units (DMU). A basic assumption of DEA for the measurement of the total technical efficiency of a DMU is that of constant returns to scale (CRS). This was later modified by Banker, Charnes and Cooper (1984) to become variable returns to scale (VRS) (Demir, 2014). According to Zeidan *et al*., (2016), Data envelopment analysis is a non-parametric technique for evaluating and measuring the relative efficiency of decision making units characterized by multiple inputs and multiple outputs.

The basic DEA works with crisp values for both the input and output values. Being a very responsive method, its efficiency is easily affected by errors bothering on imprecise data, incomplete data, judgment data, forecasting data or ambiguous data. In general, imprecise data can be presented in form of fuzzy numbers. It is therefore worthwhile to study how to evaluate the efficiency of a set of data in fuzzy form. In such a situation, FDEA becomes a useful method to overcome the shortcomings of basic DEA.

Wang *et al.* (2005) studied how to conduct efficiency assessment in interval and/or fuzzy input-output environments in a simple, rational and effective way using data envelopment analysis. They constructed a new pair of interval DEA models on the basis of interval arithmetic, which differs from the existing DEA models handling interval data.

Demir (2014) compared classical DEA and FDEA based on α-intercept method by means of an application for educational researches. He compared the relative activities of 25 high schools in the 2012-2013 education year by means of DEA and FDEA and strongly recommends that fuzzy theory be practiced for DEA problems with uncertain data in order to get more secure results in activity measurements.

Zeidan, *et al*. (2016) presented a technique to improve a statistical method based on arithmetic operations to solve fuzzy data envelopment analysis models. They transformed the original data into interval data in the form of lower and upper frontier data and used them to obtain the interval DEA efficiency scores. Their method requires that data should be distributed as a normal distribution. Thus, the technique assumes that the variables are normally distributed. This position is however at variance with the fact that DEA, being a non-parametric technique, does not assume any specific functional form relating inputs to outputs (Zhu, 2002).

Mahmudah and Lola (2016) applied the fuzzy DEA approach to measure the Indonesian universities performances under imprecise inputs and outputs. Their empirical results show that 36% of universities perform efficiently under the constant returns to scale model. For the variable returns to scale model, 52% of the universities were efficient. They discovered that the well-known universities obtained relatively low scores indicating the need for them to improve their performances in publishing scientific work in addition to providing useful information to the public through the official websites. They concluded that the results of the VRS model are better than the CRS model for both the DEA and FDEA methods.

Tlig and Hamed (2017) accessed the efficiency of commercial Tunisian Banks using two approaches of fuzzy data envelopment analysis, namely, the possibility approach and the approach based on relations between fuzzy numbers (BRONF). They evaluated the efficiency of the banks in terms of several crisp and imprecise data. Their results indicate that in a competitive environment, no-financial inputs and outputs should be considered in order to have credible and realistic efficiency scores.

Gökşen *et al*. (2015) used Data Envelopment Analysis to determine the performance levels of departments in Dokuz Eylul University (Turkey). Their study discussed the technical scores and scale scores of departments and revealed the main cause of inefficiency. The input and output goals of departments were fixed for a better efficiency.

Fatimah and Mahmudah (2017) performed a two-stage DEA for the purpose of measuring the efficiency of elementary schools in Indonesia in the period 2014/2015 using 34 DMUs. Their results show that VRS model gives better results than CRS model in the first stage. They further showed that 12 provinces in Indonesia have efficient elementary schools under the CRS model, while 17 provinces have efficient elementary schools under the VRS model. Their study established that three environmental variables; the repetition rate, the average of science of national exam and the qualified teacher's rate influence the efficiency of elementary schools in Indonesia.

Karimi (2019) analysed the technical efficiency of elementary schools in all 33 districts of Rajasthan, India from 2014 to 2016 using Data envelopment analysis VRS model. The result showed high average technical efficiency in 2016 as against 2014 and 2015. The paper further provided evidence that some high literacy rate but low technical efficiency scores were found after comparing literacy rates and technical efficiency scores of the districts, indicating that high literacy rate does not necessarily mean that districts are technically efficient.

The rest of the paper is organized as follows: Basic models of DEA and (FDEA) fuzzy Data Envelopment Analysis models, and the suggested modification to the fuzzy (interval) DEA, are discussed in the second, third and fourth sections covering the theoretical aspect of the study. Section five deals with the application of the modified fuzzy DEA model and its comparison with the model by Wang *et al*., (2005). Section six presents the summary of results and concludes the work.

Basic Models of Data Envelopment Analysis

Many authors have studied the technique of data envelopment analysis. Originally, DEA was designed to measure the relative efficiency of non-for-profit organizations. Due to its ability to model multiple input and multiple output relationships without a priori underlying functional form assumption, data envelopment analysis has also been applied to other areas which are profit oriented (Zhu, 2003). Development of new methods and models have evolved due to wide application. This paper will however, present only Charnes, Cooper and Rhodes (CCR) and Banker, Charnes and Cooper (BCC) DEA models for the purpose of understanding the fundamentals of DEA.

Charnes, Cooper, and Rhodes DEA model

The CCR DEA model by Charnes *et al*. (1978) is given below in fractional form.

y_{ro}: The number of the output by the DMU, o

x_{io}: The amount of the input used by the DMU, o

u_r: The weight of the output, r

v_i: The weight of the input, i

Objective function:

$$\max h_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$

Subject to:

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1; \quad j = 1, 2, \cdots, n$$

$$u_r, v_i \geq 0; \quad r = 1, 2, \cdots, s; \quad i\ 1,2, \cdots, m \qquad (1)$$

Transformation of fractional CCR DEA model (1) into linear form:

Objective function:

$$\max h_o = \sum_{r=1}^{s} u_r y_{ro}$$

Subject to:

$$\sum_{i=1}^{m} v_i x_{io} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0; \quad j = 1,2, \cdots, n$$

$u_r, v_i \geq 0; \quad r = 1, 2, \cdots, s; \quad i = 1,2, \cdots, m \qquad (2)$

Efficiency Frontier of the CCR DEA Model

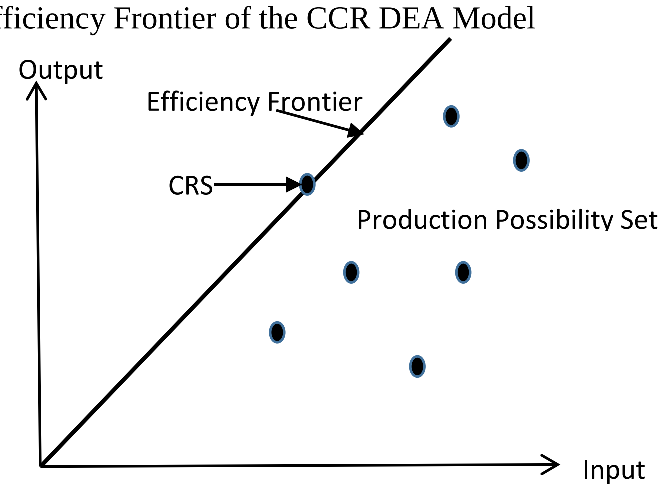

Fig. 1: Efficiency Frontier of the CCR Model

The Banker, Charnes and Cooper DEA Model

The BCC model was introduced by Banker, Charnes and Cooper in 1984. It is an extension of the CCR model. The major difference between the two models lies in the establishment of returns to scale. While constant returns to scale is assumed in CCR which means that increase in inputs results to commensurate increase in outputs, variable returns to scale is assumed in BCC implying that increase in inputs does not result to commensurate increase in outputs. Accordingly, the BCC model is more robust than the CCR model (Zeidan *et al*., 2016). The CCR and BCC radial models are depicted pictorially in Fig. 1 and Fig. 2, respectively.

The BCC model in fractional form differs from the CCR model (1) by an additional variable as presented below:

Objective function:

$$\max h_o = \frac{\sum_{r=1}^{s} u_r y_{ro} - c_o}{\sum_{i=1}^{m} v_i x_{io}}$$

Subject to:

$$\frac{\sum_{r=1}^{s} u_r y_{rj} - c_o}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1; \quad j = 1, 2, \cdots, n$$

$$u_r, v_i \geq 0; \quad r = 1, 2, \cdots, s; \quad i = 1,2, \cdots, m$$
$$c_o \;\; unrestricted\ in\ sig \qquad (3)$$

Where the new variable c_o separates scale efficiency from technical efficiency in CCR model.

Model (3) can be transformed into linear form as follows:

$$\max h_o = \sum_{r=1}^{s} u_r y_{ro} - c_o$$
$$subject\ to{:} \quad \sum_{i=1}^{m} v_i x_{io} = 1$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - c_o \leq 0; \quad j = 1, 2, \cdots, n$$
$$u_r, v_i \geq 0; \quad r = 1, 2, \cdots, s; \quad i = 1,2, \cdots, m$$
$$c_o \;\; unrestricted\ in\ sign \qquad (4)$$

If $(c_o = 0)$, then constant returns to scale (CRS) is implied.
If $(c_o < 0)$, then increasing returns to scale (IRS) is implied.
If $(c_o > 0)$, then decreasing returns to scale (DRS) is implied, (Zeidan *et al*., 2016).

Efficiency Frontier of the BCC DEA Model

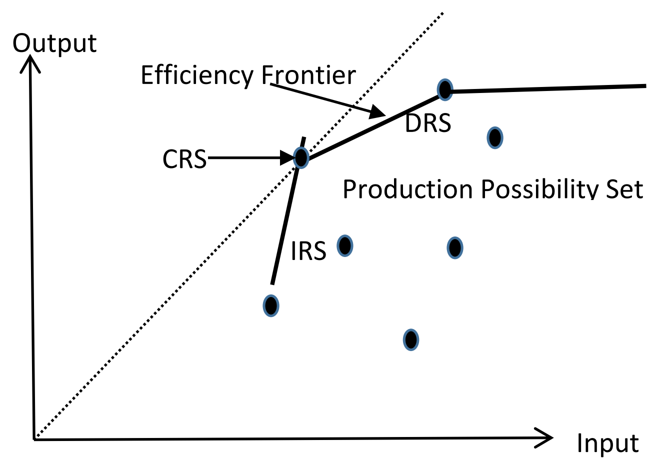

Fig. 2: Efficiency Frontier of the BCC Model

Fuzzy Data Envelopment Analysis (FDEA)

The works of Sengupta (1992, 1993) incorporated fuzziness into DEA. He suggested two membership functions, "Linear Membership Function" and "Non-linear Membership Function", for fuzzy mathematical programing model (Demir, 2014).

The approach to change fuzzy data into offset data using α-level mass to create a solution that could take advantage of a family of classical DEA models was made by Kao and Liu (2000, 2003). Leveraging on the approach, Saati *et al.*, (2002) made fuzzy CCR model as an offset programming model through defining it as programming problem using α-level. An improvement on interval data DEA was made by Wang *et al.*, (2005) by employing DEA technique in the offset data and established a fuzzy efficiency measurement.

Cooper *et al.*, (1999) created interval data envelopment analysis model (IDEA). The IDEA model can change the non-linear programing problems into linear programing problem through scale conversions and variable changes (Demir, 2014). Using Wang *et al*'*s* technique, interval data programing model can be solved like a definitive linear programming model for each DMU and an efficiency score can be made by means of each α-level (Deniz, 2009).

The technique of DMU with fuzzy data which could convert FDEA model into certain DEA model series was improved by Kao and Liu (2000).

Existing FDEA Linear Programing Formulation

Given that all inputs and outputs are incomplete as a result of uncertainties, let these values be known as $x_{ij}^L > 0$ and $y_{rj}^L > 0$ and $[x_{ij}^L, x_{ij}^U]$ and $[y_{rj}^L, y_{rj}^U]$ and they are between these top-down limits. To deal with such uncertain situation, Kao and Liu, 2000, Wang *et al.*, 2005, Güneş, 2006, and Demir, 2014 defined FDEA model with fuzzy interval data in which limited data is used for efficiency measurement to generate upper and lower bounds for each DMU, as follows:

Upper Bound

$$Max\ h_0^U = \frac{\sum_{r=1}^{s} u_r y_{r0}^U}{\sum_{i=1}^{m} v_i x_{i0}^L}$$

$$subject\ to{:}\quad \frac{\sum_{r=1}^{s} u_r y_{rj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L} \le 1;\ \ j = 1,2,\cdots,n$$

$$u_r, v_i \ge 0;\ \ r = 1,2,\cdots,s;\ \ i = 1,2,\cdots,m \qquad (5)$$

Lower Bound

$$Max\ h_0^L = \frac{\sum_{r=1}^{s} u_r y_{r0}^L}{\sum_{i=1}^{m} v_i x_{i0}^U}$$

$$subject\ to{:}\quad \frac{\sum_{r=1}^{s} u_r y_{rj}^L}{\sum_{i=1}^{m} v_i x_{ij}^U} \le 1;\ \ j = 1,2,\cdots,n$$

$u_r, v_i \geq 0;\ \ r = 1,2,\cdots,s;\ \ i = 1,2,\cdots,m$ (6)

Observe that, in the fractional programing model (5), a mixture of the upper output values and lower input values were used to obtain the upper bound of the best possible relative efficiency of DMUo, h_0^U. Similarly, for model (6), a mixture of the lower output values and upper input values were used to obtain the lower bound of the best possible relative efficiency of DMUo, h_0^L. However, the ratio of upper and lower bound values cannot logically give rise to h_0^U, neither can the ratio of lower and upper bound values logically give rise to h_0^L. Hence the need for a modification.

THE SUGGESTED MODIFICATION TO FUZZY (INTERVAL) DEA MODEL

In this study, h_0^U, the upper bound of the best possible relative efficiency of DMUo is obtained by using the ratio of upper bound values for both the output and input interval data. For h_0^L, the lower bound of the best possible relative efficiency of DMUo is obtained by using the ratio of lower bound values for both the output and input interval data. Models (5) and (6) are therefore modified as follows:

$$Max\ h_0^U = \frac{\sum_{r=1}^{s} u_r y_{r0}^U}{\sum_{i=1}^{m} v_i x_{i0}^U}$$

$$subject\ to\text{:}\ \ \frac{\sum_{r=1}^{s} u_r y_{rj}^U}{\sum_{i=1}^{m} v_i x_{ij}^U} \leq 1;\ \ j = 1,2,\cdots,n$$

$u_r, v_i \geq 0;\ \ r = 1,2,\cdots,s;\ \ i = 1,2,\cdots,m$ (7)

$$Max\ h_0^L = \frac{\sum_{r=1}^{s} u_r y_{r0}^L}{\sum_{i=1}^{m} v_i x_{i0}^L}$$

$$subject\ to\text{:}\ \ \frac{\sum_{r=1}^{s} u_r y_{rj}^L}{\sum_{i=1}^{m} v_i x_{ij}^L} \leq 1;\ \ j = 1,2,\cdots,n$$

$u_r, v_i \geq 0;\ \ r = 1,2,\cdots,s;\ \ i = 1,2,\cdots,m$ (8)

In linear programming form, models (7) and (8) become:

$$Max\ h_0^U = \sum_{r=1}^{s} u_r y_{r0}^U$$

$$Subject\ to\text{:}\ \sum_{i=1}^{m} v_i x_{i0}^U = 1$$

$$\sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} x_{ij}^U \leq 0;\ \ j = 1,2,\cdots,n$$

$u_r, v_i \geq 0;\ \ r = 1,2,\cdots,s;\ \ i = 1,2,\cdots,m$ (9)

$$Max\ h_0^L = \sum_{r=1}^{s} u_r y_{r0}^L$$

$$Subject\ to\text{:}\ \sum_{i=1}^{m} v_i x_{i0}^L = 1$$

$$\sum_{r=1}^{s} u_r y_{rj}^L - \sum_{i=1}^{m} x_{ij}^L \leq 0;\ \ j = 1,2,\cdots,n$$

$u_r, v_i \geq 0;\ \ r = 1,2,\cdots,s;\ \ i = 1,2,\cdots,m$ (10)

Application and Comparison of Classical DEA Models, existing Fuzzy Models and the Modified Models

Models (9) and (10) will be solved by first transforming the crisp data into interval data using the approach of Demir (2014). Standard errors for each variable will be added to obtain the upper frontier data, while standard errors for each variable will be subtracted to obtain the lower frontier data. For the upper frontier efficiency scores, the upper frontier values of both the output and input data will be used. To obtain the lower frontier efficiency scores, the lower frontier values of both the output and input data will be used.

To evaluate and compare results from classical DEA models, existing interval DEA models and the modified interval DEA models; real data set of 25 high schools in the 2012 – 2013 education year is taken from Demir (2014). The data description is as follows: inputs (numbers of students, teachers and classes), outputs (Transition to Higher Education Examination (YGS), Undergraduate Placement Exam (LYS) success (placement) rates, YGS point averages, all points of the LYS Maths-Science (MS), Turkish-Maths (TM), and Turkish-Social (TS) Sciences (Zeiden *et al*., 2016). See Appendix 1. The DEA models are solved using DEA-SOLVER-LV8.

The efficiency values for the classical DEA input oriented CCR and BCC models are presented in Tables 1 and 2.

Table 1: Efficiency scores for classical CCR DEA model

No.	DMU	Score	Rank		Reference(Lambda)				
1	S1	0.9701	5	S11	0.776	S17	1.552		
2	S2	0.4515	13	S11	0.563	S17	1.195		
3	S3	1	1	S3	1				
4	S4	0.4737	12	S11	0.969	S17	0.59		
5	S5	0.3693	17	S3	0.3	S11	0.258	S17	0.807
6	S6	0.5488	10	S3	0.158	S11	0.586	S17	0.571
7	S7	0.5404	11	S3	0.292	S17	0.824		
8	S8	1	1	S8	1				
9	S9	0.8642	7	S17	1.344				
10	S10	0.6864	8	S3	0.444	S8	0.17	S17	0.495
11	S11	1	1	S11	1				
12	S12	0.2179	23	S3	0.297	S17	0.657		
13	S13	0.1259	25	S3	0.136	S11	0.543	S17	0.297
14	S14	0.2196	22	S11	0.179	S17	0.882		
15	S15	0.2727	20	S11	0.059	S17	1.041		
16	S16	0.2097	24	S3	0.162	S11	0.093	S17	0.693
17	S17	1	1	S17	1				
18	S18	0.9395	6	S17	1.044				
19	S19	0.392	15	S3	0.039	S17	0.964		
20	S20	0.4203	14	S17	1.074				
21	S21	0.2379	21	S11	0.564	S17	0.468		
22	S22	0.278	19	S3	0.057	S11	0.1	S17	0.802
23	S23	0.6759	9	S11	0.341	S17	0.782		
24	S24	0.362	18	S3	0.599	S17	0.504		
25	S25	0.3865	16	S3	0.527	S17	0.336		

Table 2: Efficiency scores for classical BCC DEA model

No.	DMU	Score	Rank		Reference(Lambda)								
1	S1	1	1	S1	1								
2	S2	1	8	S2	1								
3	S3	1	1	S3	1								
4	S4	0.5106	13	S1	0.399	S3	0.366	S11	0.235				
5	S5	0.4067	15	S1	0.249	S3	0.489	S8	0.122	S17	0.139		
6	S6	0.7185	11	S1	0.199	S3	0.575	S11	0.226				
7	S7	0.5732	12	S1	0.046	S3	0.308	S8	0.189	S9	0.01	S17	0.447
8	S8	1	1	S8	1								
9	S9	1	1	S9	1								
10	S10	1	8	S10	1								
11	S11	1	1	S11	1								
12	S12	0.2221	22	S3	0.201	S11	0.095	S17	0.704				
13	S13	0.1272	25	S3	0.085	S11	0.597	S17	0.318				
14	S14	0.2203	23	S1	0.046	S11	0.143	S17	0.811				
15	S15	0.2742	20	S1	0.075	S17	0.925						
16	S16	0.2143	24	S3	0.055	S11	0.199	S17	0.746				
17	S17	1	1	S17	1								
18	S18	1	1	S18	1								
19	S19	0.3925	17	S3	0.039	S8	0.006	S17	0.954				
20	S20	0.4483	14	S8	0.328	S17	0.672						
21	S21	0.2392	21	S1	0.022	S3	0.024	S11	0.527	S17	0.427		
22	S22	0.2851	19	S11	0.158	S17	0.842						
23	S23	0.8265	10	S3	0.513	S17	0.487						
24	S24	0.3815	18	S1	0.045	S3	0.646	S8	0.116	S17	0.192		
25	S25	0.3934	16	S3	0.239	S11	0.551	S17	0.21				

Thorough examination of Tables 1 and 2 indicate that twenty-one units are inefficient with only four units efficient for the classical CCR model. For the classical BCC model however, sixteen units are inefficient while nine units are efficient. The most efficient units have efficiency score of 1 with equivalent ranking score of 1. These efficient units also serve as reference sets to the inefficient units. The most inefficient unit, S13, has least efficiency score of 0.1272 and a ranking score of 25. It has efficient units S3, S11 and S17 as reference set (Lambda). In order words, it should emulate what these efficient units are doing in order to become efficient. Notice that each efficient unit serves as its own reference (Lambda).

Table 3: Lower bound efficiency scores for the modified CCR model

No.	DMU	Score	Rank		Reference(Lambda)				
1	S1	0.7789	6	S11	1.053	S17	1.35		
2	S2	0.338	13	S11	0.807	S17	0.988		
3	S3	1	1	S3	1				
4	S4	0.3607	12	S11	1.064	S17	0.528		
5	S5	0.2816	17	S3	0.312	S11	0.409	S17	0.658
6	S6	0.4368	11	S3	0.161	S11	0.671	S17	0.5
7	S7	0.4384	10	S3	0.399	S11	0.014	S17	0.654
8	S8	0.8131	5	S3	0.074	S17	1.38		
9	S9	0.601	7	S17	1.367				
10	S10	0.5172	9	S3	0.331	S17	1.058		
11	S11	1	1	S11	1				
12	S12	0.1634	22	S3	0.246	S11	0.192	S17	0.537
13	S13	0.0898	25	S3	0.107	S11	0.616	S17	0.266
14	S14	0.1569	23	S11	0.365	S17	0.688		
15	S15	0.1979	20	S11	0.279	S17	0.812		
16	S16	0.1556	24	S3	0.044	S11	0.365	S17	0.595
17	S17	1	1	S17	1				
18	S18	0.8325	4	S17	1.045				
19	S19	0.2997	15	S3	0.141	S11	0.068	S17	0.741
20	S20	0.2525	18	S3	0.037	S17	1.022		
21	S21	0.1723	21	S11	0.635	S17	0.394		
22	S22	0.2059	19	S3	0.076	S11	0.238	S17	0.63
23	S23	0.5264	8	S11	0.362	S17	0.768		
24	S24	0.2848	16	S3	0.675	S11	0.017	S17	0.378
25	S25	0.3021	14	S11	1.141				

Table 4: Upper bound CCR efficiency scores

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	DMU	Score	Input 1 {I}{V}	Input 2 {I}{V}	Input 3 {I}{V}	Outpu 1 {O}{V	Outpu 2 {O}{V	Outpu 3 {O}{V	Outpu 4 {O}{V	Outpu 5 {O}{V	Benchmarks	{S} Input 1 {I}	{S} Input 2 {I}	{S} Input 3 {I}	{S} Outpu 1 {O}	{S} Outpu 2 {O}	{S} Outpu 3 {O}	{S} Outpu 4 {O}	{S} Outpu 5 {O}
1	F1	72,38%	0,22	0,00	0,78	0,00	0,00	1,00	0,00	0,00	11 (0,98) 17 (1,25)	0,00	23,76	0,00	51,89	74,81	0,00	75,97	89,55
2	F2	31,99%	0,22	0,00	0,78	0,00	0,00	1,00	0,00	0,00	11 (0,76) 17 (0,93)	0,00	56,62	0,00	11,85	4,80	0,00	53,06	25,67
3	F3	100,00%	0,62	0,00	0,38	1,00	0,00	0,00	0,00	0,00	12								
4	F4	34,46%	0,26	0,00	0,74	0,00	0,00	1,00	0,00	0,00	11 (1,02) 17 (0,50)	0,00	02,81	0,00	22,50	0,20	0,00	53,58	96,27
5	F5	27,41%	0,19	0,00	0,81	0,00	0,89	0,11	0,00	0,00	3 (0,33) 11 (0,37) 17 (0,63)	0,00	51,50	0,00	12,82	0,00	0,00	30,94	63,18
6	F6	42,59%	0,21	0,00	0,79	0,00	0,88	0,12	0,00	0,00	3 (0,18) 11 (0,63) 17 (0,48)	0,00	21,00	0,00	2,60	0,00	0,00	9,67	75,35
7	F7	43,59%	0,16	0,00	0,84	0,00	0,89	0,11	0,00	0,00	3 (0,40) 11 (0,01) 17 (0,65)	0,00	26,59	0,00	18,01	0,00	0,00	26,43	17,07
8	F8	76,47%	0,45	0,00	0,55	1,00	0,00	0,00	0,00	0,00	3 (0,07) 17 (1,30)	0,00	01,69	0,00	0,00	19,05	54,01	36,33	02,68
9	F9	58,31%	1,00	0,00	0,00	0,00	1,00	0,00	0,00	0,00	17 (1,33)	0,00	55,53	0,98	1,18	0,00	47,44	26,40	54,24
10	F10	49,43%	0,53	0,00	0,47	1,00	0,00	0,00	0,00	0,00	3 (0,32) 17 (1,01)	0,00	19,49	0,00	0,00	24,04	34,31	53,13	48,77
11	F11	100,00%	0,00	1,00	0,00	0,91	0,00	0,09	0,00	0,00	18								
12	F12	16,37%	0,13	0,00	0,87	0,00	0,73	0,00	0,00	0,27	3 (0,26) 11 (0,18) 17 (0,53)	0,00	68,71	0,00	23,10	0,00	8,76	7,15	0,00
13	F13	8,99%	0,17	0,00	0,83	0,00	0,74	0,00	0,00	0,26	3 (0,11) 11 (0,61) 17 (0,26)	0,00	55,04	0,00	28,02	0,00	1,99	11,67	0,00
14	F14	15,60%	0,20	0,00	0,80	0,00	0,00	1,00	0,00	0,00	11 (0,36) 17 (0,68)	0,00	83,89	0,00	29,50	5,10	0,00	23,59	11,30
15	F15	19,58%	0,17	0,00	0,83	0,00	0,00	1,00	0,00	0,00	11 (0,28) 17 (0,80)	0,00	54,75	0,00	24,36	8,90	0,00	26,41	10,44
16	F16	15,56%	0,13	0,00	0,87	0,00	0,73	0,00	0,00	0,27	3 (0,04) 11 (0,37) 17 (0,60)	0,00	55,72	0,00	24,38	0,00	9,16	4,56	0,00
17	F17	100,00%	0,67	0,33	0,00	0,26	0,00	0,00	0,74	0,00	21								
18	F18	83,12%	1,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	17 (1,04)	0,00	43,92	2,27	14,19	16,32	4,48	27,69	0,00
19	F19	30,09%	0,10	0,00	0,90	0,00	0,72	0,00	0,00	0,28	3 (0,16) 11 (0,05) 17 (0,74)	0,00	19,97	0,00	16,43	0,00	2,25	6,65	0,00
20	F20	25,12%	0,12	0,00	0,88	0,00	0,28	0,00	0,72	0,00	3 (0,03) 11 (0,01) 17 (1,02)	0,00	32,45	0,00	7,80	0,00	22,90	0,00	22,60
21	F21	17,17%	0,02	0,00	0,98	0,00	0,00	0,01	0,00	0,99	11 (0,63) 17 (0,39)	0,00	73,79	0,00	34,12	14,91	0,00	23,95	0,00
22	F22	20,71%	0,16	0,00	0,84	0,00	0,88	0,12	0,00	0,00	3 (0,07) 11 (0,24) 17 (0,63)	0,00	34,24	0,00	16,80	0,00	0,00	1,61	2,07
23	F23	52,41%	0,01	0,00	0,99	0,00	0,00	0,00	0,00	1,00	11 (0,36) 17 (0,76)	0,00	6,43	0,00	26,33	51,86	19,70	48,45	0,00
24	F24	28,32%	0,20	0,00	0,80	0,00	0,90	0,10	0,00	0,00	3 (0,68) 11 (0,01) 17 (0,37)	0,00	18,46	0,00	21,10	0,00	0,00	9,01	59,44
25	F25	30,04%	0,00	0,00	1,00	0,00	0,00	0,00	0,00	1,00	11 (1,13)	13,79	83,01	0,00	16,84	6,02	63,92	33,59	0,00

Source: Demir 2014

In Table 3, the efficient DMUs due to the modified FDEA lower bound CCR model are the same as the efficient DMUs due to the FDEA upper bound CCR model in Table 4 from Demir (2014).

Table 5: Lower bound efficiency scores for the modified BCC model

No.	DMU	Score	Rank		Reference(Lambda)						
1	S1	1	1	S1	1						
2	S2	1	1	S2	1						
3	S3	1	1	S3	1						
4	S4	0.458	13	S1	0.399	S3	0.366	S11	0.235		
5	S5	0.3369	14	S1	0.248	S3	0.512	S8	0.091	S17	0.149
6	S6	0.6481	11	S1	0.199	S3	0.575	S11	0.226		
7	S7	0.4726	12	S1	0.043	S3	0.381	S8	0.099	S17	0.477
8	S8	1	1	S8	1						
9	S9	1	1	S9	1						
10	S10	1	8	S10	1						
11	S11	1	1	S11	1						
12	S12	0.1641	22	S3	0.198	S11	0.245	S17	0.557		
13	S13	0.09	25	S3	0.085	S11	0.641	S17	0.275		
14	S14	0.1608	23	S1	0.038	S11	0.323	S17	0.639		
15	S15	0.2073	20	S1	0.064	S3	0.026	S11	0.18	S17	0.731
16	S16	0.1558	24	S3	0.053	S11	0.348	S17	0.599		
17	S17	1	1	S17	1						
18	S18	1	1	S18	1						
19	S19	0.3025	17	S3	0.041	S11	0.174	S17	0.785		
20	S20	0.2935	18	S3	0.021	S8	0.282	S17	0.697		
21	S21	0.178	21	S1	0.015	S3	0.078	S11	0.554	S17	0.353
22	S22	0.2111	19	S11	0.326	S17	0.674				
23	S23	0.7498	10	S3	0.513	S17	0.487				
24	S24	0.3033	16	S1	0.044	S3	0.672	S8	0.081	S17	0.203
25	S25	0.3108	15	S3	0.239	S11	0.551	S17	0.21		

Table 6: Upper bound BCC efficiency scores

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	DMU	Score	Input 1 {I}{V}	Input 2 {I}{V}	Input 3 {I}{V}	Outpu 1 {O}{V	Outpu 2 {O}{V	Outpu 3 {O}{V	Outpu 4 {O}{V	Outpu 5 {O}{V	Benchmarks	{S} Input 1 {I}	{S} Input 2 {I}	{S} Input 3 {I}	{S} Outpu 1 {O}	{S} Outpu 2 {O}	{S} Outpu 3 {O}	{S} Outpu 4 {O}	{S} Outpu 5 {O}
1	F1	100,00%	0,14	0,08	0,79	0,00	0,00	0,36	0,00	0,64	8								
2	F2	100,00%	0,08	0,90	0,02	0,00	0,00	0,05	0,95	0,00	0								
3	F3	100,00%	0,01	0,01	0,98	0,74	0,00	0,00	0,00	0,26	14								
4	F4	45,80%	0,00	0,00	1,00	0,00	0,36	0,64	0,00	0,00	1 (0,40) 3 (0,37) 11 (0,23)	1,70	40,90	0,00	7,85	0,00	0,00	6,19	38,61
5	F5	33,69%	0,69	0,00	0,31	0,00	0,82	0,18	0,00	0,00	1 (0,25) 3 (0,51) 8 (0,09) 17 (0,15)	0,00	1,02	0,00	5,37	0,00	0,00	1,30	18,07
6	F6	64,81%	0,00	1,00	0,00	0,00	0,00	0,48	0,52	0,00	1 (0,20) 3 (0,57) 11 (0,23)	2,84	0,00	1,95	2,88	14,93	0,00	0,00	46,04
7	F7	47,27%	0,66	0,00	0,34	0,00	0,82	0,18	0,00	0,00	1 (0,04) 3 (0,38) 8 (0,10) 17 (0,48)	0,00	11,51	0,00	16,92	0,00	0,00	20,39	3,56
8	F8	100,00%	1,00	0,00	0,00	0,18	0,57	0,25	0,00	0,00	4								
9	F9	100,00%	1,00	0,00	0,00	0,00	0,60	0,00	0,00	0,40	0								
10	F10	100,00%	1,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	0								
11	F11	100,00%	0,00	1,00	0,00	1,00	0,00	0,00	0,00	0,00	11								
12	F12	16,41%	0,17	0,00	0,83	0,00	1,00	0,00	0,00	0,00	3 (0,20) 11 (0,24) 17 (0,56)	0,00	72,86	0,00	22,90	0,00	12,59	7,87	1,81
13	F13	9,00%	0,23	0,00	0,77	0,00	1,00	0,00	0,00	0,00	3 (0,08) 11 (0,64) 17 (0,27)	0,00	56,93	0,00	27,93	0,00	3,76	12,01	0,82
14	F14	16,08%	0,22	0,00	0,78	0,00	0,00	1,00	0,00	0,00	1 (0,04) 11 (0,32) 17 (0,64)	0,00	80,04	0,00	27,49	2,32	0,00	16,97	4,32
15	F15	20,73%	0,07	0,00	0,93	0,00	0,00	0,61	0,00	0,39	1 (0,06) 3 (0,03) 11 (0,18) 17 (0,73)	0,00	45,32	0,00	21,35	6,29	0,00	16,79	0,00
16	F16	15,58%	0,00	0,00	1,00	0,00	0,14	0,00	0,00	0,86	3 (0,05) 11 (0,35) 17 (0,60)	0,07	54,99	0,00	24,26	0,00	8,42	4,33	0,00
17	F17	100,00%	0,80	0,20	0,00	1,00	0,00	0,00	0,00	0,00	14								
18	F18	100,00%	1,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	0								
19	F19	30,25%	0,13	0,00	0,87	0,00	1,00	0,00	0,00	0,00	3 (0,04) 11 (0,17) 17 (0,78)	0,00	28,40	0,00	15,99	0,00	10,29	8,14	3,85
20	F20	29,35%	0,92	0,08	0,00	0,00	0,00	0,00	1,00	0,00	3 (0,02) 8 (0,28) 17 (0,70)	0,00	0,00	0,04	9,75	5,84	16,04	0,00	4,18
21	F21	17,80%	0,11	0,00	0,89	0,00	0,00	0,63	0,00	0,37	1 (0,01) 3 (0,08) 11 (0,55) 17 (0,35)	0,00	68,59	0,00	35,00	19,92	0,00	26,25	0,00
22	F22	21,11%	0,15	0,00	0,85	0,00	0,52	0,05	0,20	0,24	11 (0,33) 17 (0,67)	0,00	39,98	0,00	17,21	3,42	7,69	5,73	7,55
23	F23	74,98%	0,00	1,00	0,00	0,00	0,00	0,00	0,00	1,00	3 (0,51) 17 (0,49)	1,38	0,00	1,99	35,83	75,78	7,01	66,59	0,00
24	F24	30,33%	0,71	0,00	0,29	0,00	0,83	0,17	0,00	0,00	1 (0,04) 3 (0,67) 8 (0,08) 17 (0,20)	0,00	4,66	0,00	20,26	0,00	0,00	3,20	46,24
25	F25	31,08%	0,00	0,00	1,00	0,00	0,15	0,00	0,00	0,85	3 (0,24) 11 (0,55) 17 (0,21)	17,54	64,68	0,00	9,90	0,00	40,15	20,81	0,00

Source: Demir 2014

In Table 5, the efficient DMUs due to the modified FDEA lower bound BCC model are the same as the efficient DMUs due to the FDEA upper bound BCC model in Table 6 from Demir (2014).

Table 7: Upper bound efficiency scores for the modified CCR model

No.	DMU	Score	Rank		Reference(Lambda)						
1	S1	1	1	S1	1						
2	S2	0.5361	14	S1	0.095	S11	0.309	S17	1.198		
3	S3	1	1	S3	1						
4	S4	0.5495	12	S1	0.339	S3	0.321	S11	0.413		
5	S5	0.4389	16	S1	0.091	S3	0.383	S17	0.755		
6	S6	0.6544	10	S3	0.175	S11	0.624	S17	0.493		
7	S7	0.6257	11	S3	0.188	S17	0.971				
8	S8	1	1	S8	1						
9	S9	0.9668	7	S8	0.609	S17	0.528				
10	S10	0.7613	9	S3	0.435	S8	0.18	S17	0.49		
11	S11	1	1	S11	1						
12	S12	0.275	22	S3	0.165	S17	0.853				
13	S13	0.1596	25	S1	0.016	S3	0.146	S11	0.488	S17	0.309
14	S14	0.2734	23	S1	0.027	S17	1.005				
15	S15	0.3584	19	S17	1.097						
16	S16	0.2611	24	S3	0.125	S17	0.844				
17	S17	1	1	S17	1						
18	S18	0.9739	6	S17	1.043						
19	S19	0.4919	15	S8	0.155	S17	0.817				
20	S20	0.5371	13	S17	1.071						
21	S21	0.2925	21	S1	0.033	S11	0.481	S17	0.478		
22	S22	0.3427	20	S3	0.021	S17	0.964				
23	S23	0.7697	8	S17	1.058						
24	S24	0.4312	18	S3	0.515	S17	0.623				
25	S25	0.4382	17	S3	0.831						

Table 8: Lower bound CCR efficiency scores

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	DMU	Score	Input 1 {I}{V}	Input 2 {I}{V}	Input 3 {I}{V}	Outpu 1 {O}{V	Outpu 2 {O}{V	Outpu 3 {O}{V	Outpu 4 {O}{V	Outpu 5 {O}{V	Benchmarks	{S} Input 1 {I}	{S} Input 2 {I}	{S} Input 3 {I}	{S} Outpu 1 {O}	{S} Outpu 2 {O}	{S} Outpu 3 {O}	{S} Outpu 4 {O}	{S} Outpu 5 {O}
1	F1	100,00%	0,30	0,00	0,70	0,00	0,00	1,00	0,00	0,00	7								
2	F2	56,58%	0,52	0,16	0,32	0,23	0,00	0,77	0,00	0,00	1 (0,08) 3 (0,04) 11 (0,30) 17 (1,28)	0,00	0,00	0,00	0,00	8,99	0,00	47,69	43,55
3	F3	100,00%	0,90	0,00	0,10	1,00	0,00	0,00	0,00	0,00	12								
4	F4	55,27%	0,05	0,00	0,95	0,00	0,36	0,64	0,00	0,00	1 (0,34) 3 (0,32) 11 (0,42)	0,00	45,84	0,00	11,29	0,00	0,00	14,35	44,62
5	F5	44,73%	0,52	0,00	0,48	0,00	0,66	0,34	0,00	0,00	1 (0,10) 3 (0,38) 17 (0,75)	0,00	18,32	0,00	3,65	0,00	0,00	17,83	61,64
6	F6	67,09%	0,30	0,70	0,00	0,00	0,82	0,18	0,00	0,00	3 (0,16) 11 (0,65) 17 (0,53)	0,00	0,00	0,54	1,29	0,00	0,00	11,80	80,89
7	F7	63,36%	0,68	0,00	0,32	0,00	0,97	0,03	0,00	0,00	3 (0,24) 8 (0,20) 17 (0,63)	0,00	0,93	0,00	14,51	0,00	0,00	22,77	12,36
8	F8	100,00%	0,98	0,00	0,02	0,21	0,79	0,00	0,00	0,00	7								
9	F9	97,74%	1,00	0,00	0,00	0,00	0,89	0,00	0,00	0,11	8 (0,63) 17 (0,51)	0,00	22,57	3,78	3,75	0,00	20,13	15,45	0,00
10	F10	77,09%	1,00	0,00	0,00	0,51	0,04	0,46	0,00	0,00	3 (0,46) 8 (0,15) 17 (0,51)	0,00	25,47	2,77	0,00	0,00	0,00	29,07	2,92
11	F11	100,00%	0,00	1,00	0,00	0,23	0,00	0,77	0,00	0,00	5								
12	F12	27,51%	0,69	0,00	0,31	0,00	0,98	0,00	0,00	0,02	3 (0,20) 8 (0,13) 17 (0,63)	0,00	63,36	0,00	18,60	0,00	5,15	2,09	0,00
13	F13	15,88%	0,20	0,00	0,80	0,00	0,14	0,61	0,00	0,25	1 (0,01) 3 (0,14) 11 (0,49) 17 (0,31)	0,00	39,74	0,00	26,21	0,00	0,00	8,68	0,00
14	F14	27,21%	1,00	0,00	0,00	0,00	0,25	0,75	0,00	0,00	1 (0,10) 17 (0,82)	0,00	83,96	0,24	19,42	0,00	0,00	4,04	4,05
15	F15	35,87%	1,00	0,00	0,00	0,00	0,00	0,91	0,00	0,09	1 (0,10) 17 (0,86)	0,00	55,06	1,18	15,90	3,23	0,00	7,91	0,00
16	F16	26,05%	0,68	0,00	0,32	0,00	0,00	0,00	1,00	0,00	3 (0,13) 17 (0,84)	0,00	37,54	0,00	19,07	0,30	1,51	0,00	8,67
17	F17	100,00%	0,23	0,63	0,14	0,00	0,00	0,00	0,00	1,00	18								
18	F18	97,54%	1,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	17 (1,04)	0,00	54,93	3,28	13,93	15,68	3,78	27,36	0,00
19	F19	49,05%	1,00	0,00	0,00	0,00	0,87	0,00	0,00	0,13	8 (0,15) 17 (0,82)	0,00	0,28	0,45	12,20	0,00	4,63	2,79	0,00
20	F20	54,03%	1,00	0,00	0,00	0,00	0,44	0,00	0,56	0,00	8 (0,01) 17 (1,06)	0,00	85,19	3,77	6,71	0,00	23,51	0,00	24,85
21	F21	29,22%	0,18	0,00	0,82	0,00	0,00	0,71	0,00	0,29	1 (0,03) 11 (0,48) 17 (0,48)	0,00	68,44	0,00	30,20	13,81	0,00	17,87	0,00
22	F22	34,23%	0,67	0,00	0,33	0,00	0,00	0,00	1,00	0,00	3 (0,02) 17 (0,96)	0,00	16,11	0,00	10,81	1,86	2,30	0,00	17,55
23	F23	77,13%	0,00	1,00	0,00	0,00	0,00	0,00	0,00	1,00	17 (1,06)	3,64	0,00	1,24	16,10	36,09	3,24	29,56	0,00
24	F24	43,58%	0,72	0,02	0,26	0,00	1,00	0,00	0,00	0,00	3 (0,54) 8 (0,09) 17 (0,47)	0,00	0,00	0,00	17,59	0,00	3,11	7,57	63,01
25	F25	43,52%	0,00	0,00	1,00	0,00	0,00	0,00	0,00	1,00	3 (0,83)	24,12	29,05	0,00	17,96	24,08	22,36	35,70	0,00

Source: Demir 2014

In Table 7, the efficient DMUs due to the modified FDEA upper bound CCR model are the same as the efficient DMUs due to the FDEA lower bound CCR model in Table 8 from Demir (2014).

Table 9: Upper bound efficiency scores for the modified BCC model

No.	DMU	Score	Rank		Reference(Lambda)								
1	S1	1	1	S1	1								
2	S2	1	1	S2	1								
3	S3	1	1	S3	1								
4	S4	0.5538	13	S1	0.399	S3	0.366	S11	0.235				
5	S5	0.4635	16	S1	0.25	S3	0.474	S8	0.143	S17	0.133		
6	S6	0.7655	11	S1	0.199	S3	0.575	S11	0.226				
7	S7	0.6458	12	S1	0.047	S3	0.285	S8	0.188	S9	0.04	S17	0.44
8	S8	1	1	S8	1								
9	S9	1	1	S9	1								
10	S10	1	1	S10	1								
11	S11	1	1	S11	1								
12	S12	0.2754	22	S3	0.176	S8	0.04	S17	0.784				
13	S13	0.1615	25	S3	0.086	S11	0.557	S17	0.357				
14	S14	0.2737	23	S1	0.053	S17	0.947						
15	S15	0.3598	19	S1	0.075	S17	0.925						
16	S16	0.2654	24	S3	0.058	S11	0.062	S17	0.881				
17	S17	1	1	S17	1								
18	S18	1	1	S18	1								
19	S19	0.4931	15	S8	0.065	S17	0.935						
20	S20	0.5481	14	S8	0.328	S17	0.672						
21	S21	0.2928	21	S1	0.027	S11	0.485	S17	0.488				
22	S22	0.3464	20	S11	0.019	S17	0.981						
23	S23	0.8672	10	S3	0.513	S17	0.487						
24	S24	0.4451	18	S1	0.045	S3	0.641	S8	0.124	S17	0.19		
25	S25	0.4583	17	S3	0.239	S11	0.551	S17	0.21				

Table 10: Lower bound BCC efficiency scores

C:\Users\Emre\Desktop\ems alt sınır etkinlik (girdi üst çıktı alt).xls_VRS_RAD_IN

	DMU	Score	Input 1 {I}{V}	Input 2 {I}{V}	Input 3 {I}{V}	Outpu 1 {O}{V	Outpu 2 {O}{V	Outpu 3 {O}{V	Outpu 4 {O}{V	Outpu 5 {O}{V	Benchmarks	{S} Input 1 {I}	{S} Input 2 {I}	{S} Input 3 {I}	{S} Outpu 1 {O}	{S} Outpu 2 {O}	{S} Outpu 3 {O}	{S} Outpu 4 {O}	{S} Outpu 5 {O}
1	F1	100,00%	0,13	0,10	0,77	0,00	0,00	0,36	0,00	0,64	8								
2	F2	100,00%	0,08	0,91	0,01	0,00	0,00	0,05	0,95	0,00	0								
3	F3	100,00%	0,00	0,02	0,97	0,73	0,00	0,00	0,00	0,27	10								
4	F4	55,38%	0,00	0,00	1,00	0,00	0,36	0,64	0,00	0,00	1 (0,40) 3 (0,37) 11 (0,23)	1,09	37,53	0,00	7,85	0,00	0,00	6,19	38,61
5	F5	46,35%	0,81	0,19	0,00	0,00	0,92	0,08	0,00	0,00	1 (0,25) 3 (0,47) 8 (0,14) 17 (0,13)	0,00	0,00	0,66	4,64	0,00	0,00	0,15	16,03
6	F6	76,55%	0,00	1,00	0,00	0,00	0,00	0,47	0,53	0,00	1 (0,20) 3 (0,57) 11 (0,23)	3,73	0,00	2,63	2,88	14,93	0,00	0,00	46,04
7	F7	64,58%	0,83	0,17	0,00	0,00	0,85	0,06	0,00	0,09	1 (0,05) 3 (0,29) 8 (0,19) 9 (0,04) 17	0,00	0,00	0,18	14,80	0,00	0,00	17,17	0,00
8	F8	100,00%	0,84	0,00	0,16	0,00	0,94	0,06	0,00	0,00	6								
9	F9	100,00%	1,00	0,00	0,00	0,00	0,59	0,00	0,00	0,41	1								
10	F10	100,00%	1,00	0,00	0,00	0,00	0,00	0,00	0,00	1,00	0								
11	F11	100,00%	0,00	1,00	0,00	1,00	0,00	0,00	0,00	0,00	7								
12	F12	27,54%	0,68	0,00	0,32	0,00	1,00	0,00	0,00	0,00	3 (0,18) 8 (0,04) 17 (0,78)	0,00	73,99	0,00	17,46	0,00	9,93	3,90	9,97
13	F13	16,15%	0,23	0,00	0,77	0,00	1,00	0,00	0,00	0,00	3 (0,09) 11 (0,56) 17 (0,36)	0,00	44,84	0,00	26,24	0,00	2,91	10,92	4,14
14	F14	27,37%	1,00	0,00	0,00	0,00	0,00	1,00	0,00	0,00	1 (0,05) 17 (0,95)	0,00	85,83	0,08	20,90	4,66	0,00	13,52	17,56
15	F15	35,98%	1,00	0,00	0,00	0,00	0,00	1,00	0,00	0,00	1 (0,07) 17 (0,93)	0,00	55,92	1,10	16,65	5,58	0,00	12,70	6,83
16	F16	26,54%	0,18	0,00	0,82	0,00	1,00	0,00	0,00	0,00	3 (0,06) 11 (0,06) 17 (0,88)	0,00	41,93	0,00	18,46	0,00	5,51	0,62	11,37
17	F17	100,00%	0,39	0,00	0,61	0,00	0,00	0,00	0,00	1,00	14								
18	F18	100,00%	0,85	0,15	0,00	0,00	0,00	0,00	0,00	1,00	0								
19	F19	49,31%	1,00	0,00	0,00	0,00	1,00	0,00	0,00	0,00	8 (0,07) 17 (0,93)	0,00	12,51	0,58	11,67	0,00	8,22	4,60	8,24
20	F20	54,81%	1,00	0,00	0,00	0,00	0,00	0,00	1,00	0,00	8 (0,33) 17 (0,67)	0,00	43,12	3,37	9,59	6,80	16,08	0,00	2,64
21	F21	29,28%	0,29	0,00	0,71	0,00	0,00	1,00	0,00	0,00	1 (0,03) 11 (0,48) 17 (0,49)	0,00	69,08	0,00	30,43	14,26	0,00	18,88	1,23
22	F22	34,64%	0,16	0,00	0,84	0,00	0,30	0,01	0,69	0,00	11 (0,02) 17 (0,98)	0,00	17,42	0,00	10,77	2,90	4,45	1,27	19,62
23	F23	86,72%	0,00	1,00	0,00	0,00	0,00	0,00	0,00	1,00	3 (0,51) 17 (0,49)	1,67	0,00	2,62	35,83	75,78	7,01	66,59	0,00
24	F24	44,51%	0,82	0,18	0,00	0,00	0,93	0,07	0,00	0,00	1 (0,05) 3 (0,64) 8 (0,12) 17 (0,19)	0,00	0,00	0,56	19,67	0,00	0,00	2,26	44,58
25	F25	45,83%	0,00	0,00	1,00	0,00	0,14	0,00	0,00	0,86	3 (0,24) 11 (0,55) 17 (0,21)	25,45	63,79	0,00	9,90	0,00	40,15	20,81	0,00

Source: Demir 2014

In Table 9, the efficient DMUs due to the modified FDEA upper bound BCC model are the same as the efficient DMUs due to the FDEA lower bound BCC model in Table 10, from Demir (2014).

Results and Conclusion

Results of the efficient decision making units (DMUs) due to classical DEA models, fuzzy DEA models proposed by Wang *et al.*, (2005) and adopted by Demir (2014) and the modified fuzzy DEA models are presented in summary form in tables 11 and 1

Table 11: Comparison of DEA, FDEA and modified FDEA Efficiency Results via CCR

Classical DEA	**Lower and upper efficient FDEA (Demir)**		**Lower and upper efficient FDEA (Modified)**	
	L	U	L	U
S3	S1	S3	S3	S1
S8	S3	S11	S11	S3
S11	S8	S17	S17	S8
S17	S11			S11
	S17			S17

Table 12: Comparison of DEA, FDEA and modified FDEA Efficiency Results via BCC

Classical DEA	Lower and upper efficient FDEA (Demir)		Lower and upper efficient FDEA (Modified)	
	L	U	L	U
S1	S1	S1	S1	S1
S2	S2	S2	S2	S2
S3	S3	S3	S3	S3
S8	S8	S8	S8	S8
S9	S9	S9	S9	S9
S10	S10	S10	S10	S10
S11	S11	S11	S11	S11
S17	S17	S17	S17	S17
S18	S18	S18	S18	S18

Table 11 presents the efficient DMUs from the three DEA models. A major finding in the case of CCR, when the results of Demir and that of the modified model are compared is that, the efficient DMUs when the upper bound model (Model 5) is applied, corresponds to the efficient DMUs when the lower bound modified model (Model 10) is applied. Similarly, when the lower bound model (Model 6) is applied, the result corresponds to that of the upper bound modified model (Model 9).

The implication of this finding is that, the ambiguity created by the mixture of upper bound and lower bound values to generate efficiency scores in each of Models 5 and 6 can be avoided. Instead, the modified Models 9 and 10, where upper bound values are used exclusively to generate upper efficiency scores and lower bound values are used exclusively to generate lower efficiency scores can be adopted to avoid the ambiguity.

In the case of BCC, Table 12, the efficient DMUs are the same for all the models compared. This is not unexpected since BCC is more robust and adopts variable returns to scale (VRS) frontier as against the more restrictive CCR which adopts the constant returns to scale (CRS) frontier.

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