Modeling Volatility of Exchange Rates Returns in the Nigerian Foreign Exchange Market in the Presence of Non-Gaussian Errors

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Abstract

This study investigates volatility behaviour of exchange rates returns of Naira against CFA, Euro, Great British Pounds, US Dollar, West African Unit of Account (WAUA) and Japanese Yen in Nigeria using historical volatility approach as well as symmetric and asymmetric Autoregressive Conditional Heteroskedasticity (GARCH) models in the presence of non-Gaussian errors. The study utilizes daily quotations of these exchange rates from 12/11/2001 to 04/13/2018 making a total of 4008 observations each. Historical (annualized) volatility approach as well as symmetric GARCH (1,1) and asymmetric EGARCH (1,1) models were used to model the exchange rates return series. Results showed that CFA and USD have the highest and least annualized volatilities (market risk) respectively among the six exchange rates returns as measured by historical approach. The symmetric GARCH (1,1) model showed volatility clustering with evidence of shock persistence in the six exchange rates return series. The asymmetric EGARCH (1,1) model found evidence of asymmetry and leverage effects in the Nigerian foreign exchange market indicating that negative shocks (bad news) generate more volatility than positive shocks (good news) of similar magnitudes. All the estimated models were found to be stationary and mean reverting indicating the predictability and stability of the conditional variances of the foreign exchange rates returns. This result suggests that no matter how high or low the foreign exchange rates shall move in the exchange market, they shall eventually return to their long-run averages. Stationary and mean reverting stocks provide good and long term investment opportunities for investors.

Key words: Exchange Rate; Historical Volatility; GARCH Models; Leverage Effect; Mean Reversion, Nigeria.

Introduction

Exchange rate volatility is a measure of fluctuations in local currencies relative to foreign currencies in exchange rate markets. Foreign exchange risk is a financial risk associated with an exposure to unanticipated changes in the exchange rate between two currencies in an exchange market. For a large open economy like Nigeria, exchange rate volatility plays a crucial role in both financial transactions and international trade and is of crucial interest for policy makers in every economy (Smith *et al.*, 1990). Exchange rate volatility or fluctuation is distributive in the foreign exchange market as both buyers and sellers, traders and investors as well as importers and exporters of goods and services are all exposed to the same level of risk and uncertainty. Foreign currency's value fluctuates according to the forces of demand and supply, meaning that if demand for a particular currency decreases and supply increases, it can cause depreciation of the currency's value. On the other hand if supply decreases and demand increases, this can cause appreciation of the currency's value (Madura, 1989). When exchange rate volatility increases it leads to uncertainty in pricing which hurts traders, investors, buyers or importers who spent more on the same quantity. Exchange rate volatility and price fluctuation have significant implications on the profits and survival of any business enterprise as well as on the volume of international trade (Smith *et al*., 1990).

The Nigerian economy is very sensitive to fluctuations in the foreign exchange rates given the fact that Nigeria depends significantly on imports and uses exchange rates in the transaction; there is an increasing amount of foreign investment in the country and that important reserves are held in foreign exchange, especially the US dollars. Moreover, banks as well as other financial institutions usually invest in foreign exchange instruments. The benefits of exchange rate stability to sustainable economic development in Nigeria and any other country cannot be overemphasized. The central bank of Nigeria (CBN) and other monetary agencies, therefore, try to control and avoid wide divergence between the official exchange rate and parallel exchange rates. However, the Nigerian Naira continues to fluctuate widely against other foreign currencies in spite of these policy efforts by CBN and other Nigeria monetary authorities to maintain stable exchange rate. Exchange rate fluctuation makes investment decisions and international trade more difficult due to the uncertainty and risk associated with increases in exchange rate volatility. The amount of international publications by Nigerian researchers and academics has drastically reduced in recent times due to increasing exchange rate volatility as more Nigerian currencies are needed in exchange of a small amount of other foreign currencies. This has hindered scientific communication, findings and exchange of ideas across the globe (Kuhe and Agaigbe, 2018).

Foreign exchange rate volatility has been a useful measure of uncertainty about the financial and economic environment of any country including Nigeria. Traders and investors in foreign exchange market, financial institutions as well as Policy makers need accurate estimates about the future values of exchange rates. Since exchange rate volatility or fluctuation increases transaction costs and reduces the gains of international trade, having a good understanding of exchange rate volatility measures and forecasting using accurate volatility measuring tools is paramount and imperative for asset pricing and risk management in exchange markets (Kuhe and Agaigbe, 2018).

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model developed by Bollerslev (1986) has been found useful in modeling and forecasting volatility of financial time series data. The GARCH model is good at capturing volatility clustering, shocks persistence and other statistical regularities (stylized facts), it does not capture asymmetry and leverage effect in financial returns. This led to extension of symmetric GARCH model to asymmetric models such as the exponential GARCH (EGARCH) due to Nelson (1991), threshold GARCH (TGARCH) of Zakoian (1994), power GARCH (PGARCH) due to Ding *et al*. (1993) and Glosten, Jaganathan and Runkle GARCH (GJR-GARCH) introduced by Glosten *et al.* (1993) among others are very useful in capturing asymmetry and leverage effect in the financial return series.

Many scholars have applied both symmetric and asymmetric GARCH family models in modeling and forecasting exchange rate volatility across the globe both in developed and emerging economies and the empirical evidence are well documented in the literature. For example, Kamal *et al*. (2012) examined the performance of GARCH family models in forecasting the volatility behaviour of Pakistani FOREX market using daily FOREX rates data ranging from January 2001 to December 2009. The symmetric GARCH-M model found evidence of volatility clustering and persistence of shocks in the FOREX market. The result of the EGARCH showed evidence of asymmetry in volatility. Thorlie *et al.* (2014) examined the accuracy and forecast performance ability of exchange rate return volatility for the Leones/USA dollars in ARMA-GARCH framework with normal and non-normal distributions using monthly exchange rate returns data from January 2004 to December 2013. They found that symmetric and asymmetric GARCH models performed better under non-normal distribution than the normal distribution and improved the overall estimation process for measuring conditional volatility in exchange rate returns. The GJR-GARCH model with skewed Student-t innovation was most successful and better in forecasting the Sierra Leone exchange rate volatility. The asymmetric GARCH models found evidence of asymmetry in exchange rate returns indicating presence of leverage effect. Omari *et al*. (2017) employed GARCH-types models in modeling exchange rate volatility of the USD/KES exchange rate in Kenya using daily closing observations for the period January 3, 2003 to December 31, 2015. The study applied symmetric GARCH models such as GARCH (1, 1) and GARCH-M in capturing most of the stylized facts about exchange rate returns like volatility clustering and persistence as well as asymmetric models such as EGARCH (1, 1), GJR-GARCH (1, 1) and APARCH (1, 1) models that capture asymmetry and leverage effect. Results showed that asymmetric APARCH, GJR-GARCH and EGARCH models with Student-t innovation density were most adequate for estimating the volatility of the exchange rates in Kenya. See also Vee *et al*. (2011) and Abdalla (2012) for similar contributions.

In Nigeria, several documented evidence on exchange rate volatility modeling is also found in the literature. For example, Adeoye and Atanda (2011) applied ARCH and GARCH models to examine the degree of volatility of USD/Naira exchange rate using monthly exchange rate data from 1986 to 2008. The result showed the presence of volatility clustering and over persistence of volatility shocks. Onakoya (2013) employed asymmetric EGARCH model to investigate the relative contributions of stock market volatility on real output in Nigeria over the period from 1980 to 2010. The findings of the study revealed that volatility shock was quite persistent in Nigeria and this might distort economic growth of the country. Musa and Abubakar (2014) in their study employed GARCH (1,1), GJR-GARCH (1,1), TGARCH (1,1) and TS-GARCH (1,1) models to investigate the volatility of daily closing US Dollar/Naira exchange rate data for the period 01/06/2000 to 26/07/2011 consisting of 4083 observations. Results revealed that apart from TGARCH (1,1) model which showed mean-reverting behaviour in the conditional variance, all the other GARCH models showed over persistence of volatility shocks indicating non-stationarity of the conditional variance processes. The GJR-GARCH (1,1) and TGARCH (1,1) models showed evidence for the existence of statistically significant asymmetric effect without leverage effect. TGARCH (1,1) and TS-GARCH (1,1) models were found superior over other GARCH models in explaining USD/Naira exchange rate volatility in Nigeria.

David *et al*. (2016) examined the naira exchange rate against four foreign currencies: US Dollar, Euro, British Pound and Japanese Yen. The weekly data on these exchange rates spanned from January 2002 to May 2015. They employed lower symmetric and asymmetric GARCH specifications. Results of the symmetric models showed volatility persistence in all the foreign exchange rate returns. Results of the asymmetric model showed superior forecasting performance over symmetric GARCH with different impacts for both negative and positive volatility shocks. Emenike (2016) employed symmetric GARCH (1,1) and asymmetric GJR-GARCH (1,1) models to estimate and compare the volatilities of official, interbank and bureaux de change markets Naira/US dollar exchange rates from January 1995 to December 2014. The results of the study showed evidence of volatility clustering in the interbank market and bureaux de change Naira/US dollar exchange rates. Volatility clustering and persistence was found to be higher in bureaux de change than in other exchange rates in Nigeria. Recently, Kuhe and Agaigbe (2018) used symmetric and asymmetric GARCH models with non-Gaussian errors to study the volatility hehaviour of Naira/US Dollar exchange rate in Nigeria. The study utilized daily closing Naira/US Dollar exchange rate data from 12th November, 2001 to 12th January, 2017 with a total of 3665 observations. Results showed that symmetric ARCH (3) and basic GARCH (1,1) models with student-t innovations as well as asymmetric EGARCH (1,1) with GED distribution and TGARCH (1,1) with student-t innovation were the best fitting models for the Naira/US Dollar exchange rate log return series. All the estimated models showed high persistence of volatility shock in the conditional variance. The asymmetric EGARCH (1,1) and TGARCH (1,1) models showed supportive evidence for the existence of asymmetry and leverage effects which suggested that negative shocks produced more volatility in Nigerian foreign exchange market than positive shocks of the same magnitude.

From the foregoing, it is crystal clear that

GARCH family models have been found useful by researchers around the world, including Nigeria in measuring exchange rate volatility. It is also glaring that many researchers in Nigeria mainly applied the GARCH family models to study the volatility behaviour of Naira/US Dollar exchange rate. This creates a wide gap in the literature since Nigeria does not only transact business with the United States alone but also with other countries of the world and uses their currencies as well. This study therefore, extends the existing literature by combining the historical measure of volatility and symmetric GARCH model as well as asymmetric EGARCH model with heavy-tailed distributions in measuring exchange rate volatility of Nigeria Naira against six selected foreign currencies using more recent data.

Materials and methods

Source of Data and Data Transformation

The data used in this paper are the daily closing exchange rates of the Nigerian Naira against CFA Franc, Euro, Great British Pound Sterling (GBP), US Dollar, West African Units of Account (WAUA) and the Japanese Yen (JPY) from 12/10/2001 to 04/13/2018 making a total of 4008 observations each. The daily returns r_t are calculated as the continuously compounded log returns corresponding to the first differences in logarithms of closing exchange rates of successive days.

$$r_t = \log\left(\frac{R_t}{R_{t-1}}\right) \times 100 = [\log(R_t) - \log(R_{t-1})] \times 100 \qquad (1)$$

Where r_t denotes the daily exchange rate returns, R_t denotes the closing market index at the current day (t) and r_{t-1} denotes the closing market index at the previous day (t - 1).

Methods of Data Analysis

The following statistical tools are employed in the analysis of data in this work.

Jarque-Bera Test of Normality

Jarque and Bera (1980, 1987) proposed a normality test which is goodness-of-fit test of whether sample data have the skewness and kurtosis matching a normal distribution. The test is used to test the null hypothesis that the series is normally distributed. Given a return series $\{r_t\}$ the test statistic JB is defined as:

$$JB = \frac{T}{6}\left(S_k^2 + \frac{1}{4}(K_u - 3)^2\right) \qquad (2)$$

where S_k is the sample skewness which is estimated by:

$$S_k = \frac{\mu_3}{\mu_2^{3/2}} = T^{1/2}\sum_{t=1}^{T}(r_t - \bar{r})^3 \Big/ \left(\sum_{t=1}^{T}(r_t - \bar{r})^2\right)^{3/2} \qquad (3)$$

and S_u is the sample kurtosis which is estimated by:

$$K_u = \frac{\mu_4}{\mu_2^2} = T\sum_{t=1}^{T}(r_t - \bar{r})^4 \Big/ \left(\sum_{t=1}^{T}(r_t - \bar{r})^2\right)^{2} \qquad (4)$$

where T is the number of observations and $\bar{r}$ is the sample mean. The normal distribution has a skewness equal to 0 with a kurtosis of 3.

Augmented Dickey-Fuller (ADF) Unit Root Test

The ADF unit root test is used to test whether the given time series contains a unit root or whether the given series is stationary or not, Dickey and Fuller (1979). The Augmented Dickey-Fuller (ADF) unit root test constructs a parametric correction for higher-order correlation by assuming that the series follows an AR (p) process:

$$Y_t = \psi_1 Y_{t-1} + \psi_2 Y_{t-2} + \cdots + \psi_p Y_{t-p} + u_t \tag{5}$$

$$\Delta Y_t = \psi^* Y_{t-1} + \psi_1 \Delta Y_{t-1} + \psi_2 \Delta Y_{t-2} + \cdots + \psi_{p-1} \Delta Y_{t-p+1} + u_t \tag{6}$$

where $u_t \sim \text{iid}(0, \sigma^2)$ and $\psi^* = (\psi_1 + \psi_2 + \cdots + \psi_p) - 1$. If $\psi^* = 0$, against the alternative $\psi^* < 0$, then Y_t contains a unit root. To test the null hypothesis, the ADF test is evaluated using the t $-$statistic:

$$t_\psi = \psi^* / SE(\psi^*) \tag{7}$$

where ψ^* is the estimate of ψ, and SE(ψ^*) is the coefficient standard error. This test can be compared against the critical values at the conventional test sizes.

Measuring Historical Volatility

Volatility is a measure of the spread of outcomes of asset returns. It is associated with the sample standard deviation of returns over a given period of time. It is computed using the following equation:

$$\hat{\sigma} = \sqrt{\frac{1}{n-1} \sum_{t=1}^{n} (r_t - \bar{r})} \tag{8}$$

where $\bar{r}$ is the mean return defined by:

$$\bar{r} = \frac{1}{n} \sum_{i=1}^{n} r_i \tag{9}$$

Historical volatility is the *annualized* standard deviation of returns. The annualized volatility is given by

$$\sigma_{annualized} = \sqrt{252.\frac{1}{n} \sum_{t=1}^{n} r_t^2} \tag{10}$$

where r_t is the return of an asset over period $t, \bar{r}$ is the average return over *t* periods, 252 is the annual number of trading days and *n* is the rate of return over *t*th time interval.

Model Specification

Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Model

The symmetric GARCH model was extended by Bollerslev (1986) from the earlier work of Engle (1982). Assuming a log return series $r_t = \mu + \varepsilon_t$ where ε_t is the error term at time t. The ε_t follows a GARCH (*p, q*) model if:

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} \tag{11}$$

with constraints $\omega > 0, \alpha_i \geq 0, i = 1, 2, \ldots, q$ and $\beta_j \geq 0, j = 1,2, \ldots, p;$ $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$

1 to ensure conditional variance to be positive as well as stationary. The basic GARCH (1,1) model is given by:

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \tag{12}$$

The stationarity condition of a ba sic GARCH (1,1) model is that the sum of ARCH and GARCH terms are strictly less than one (i.e.,$\alpha_1 + \beta_1 < 1$). The GARCH (1,1) model is adequate in capturing all volatility in any financial series.

The Exponential GARCH (EGARCH) Model

The EGARCH mod el was extended by Nelson (1991) to capture asymmetry and leverage effects between positive and negative stock returns. The EGARCH (1,1) model is specified as follows:

$$\ln h_t = \omega + \alpha_1 \left|\frac{\varepsilon_{t-1}}{h_{t-1}}\right| + \gamma \left[\frac{\varepsilon_{t-1}}{h_{t-1}}\right] + \beta_1 \ln h_{t-1} \tag{13}$$

where γ denotes the asymmetry or leverage effect parameter. There is presence of asymmetry when $\gamma \neq 0$; leverage effect exists if $\gamma < 0$ indicating that bad news ($\varepsilon_{t-1} < 0$) increases volatility more than good news ($\varepsilon_{t-1} > 0$) of the same magnitude.

Estimation and Error Distributions

In order to estimate and improve the efficiency of GARCH models in modeling the returns series for high frequency financial time series, we obtain the estimates of GARCH process by maximizing the likelihood function:

$$L(\theta_{t)}) = -\,{}^{1}\!/_{2}\sum_{t-1}^{T}\left(\ln 2\pi + ln\sigma_t^2 + \frac{\varepsilon_t^2}{\sigma_t^2}\right) \tag{14}$$

where σ_t^2 is the conditional variance of returns. Two error distributions are employed to estimate volatility in this study:

(i) For student's t - distribution, the log-likelihood contributions are of the form:

$$l_t = \frac{1}{2}\log\left[\frac{\pi(v-2)\Gamma\left({}^{v}\!/_{2}\right)^2}{\Gamma\left({}^{(v+1)}\!/_{2}\right)}\right] - \frac{1}{2}\log\sigma_t^2 - \frac{(v+1)}{2}\log\left[1 + \frac{(y_t - X_t'\theta)^2}{\sigma_t^2(v-2)}\right] \tag{15}$$

where the degree of freedom $v > 2$ controls the tail behaviour. The t - distribution approaches the normal distribution as $v \rightarrow \infty$

(ii) For the GED, we have

$$l_t = -\frac{1}{2}\log\left[\frac{\Gamma\left({}^{1}\!/_{r}\right)^3}{\Gamma\left({}^{3}\!/_{r}\right)\left({}^{r}\!/_{2}\right)^2}\right] - \frac{1}{2}\log\sigma_t^2 - \left[\frac{\Gamma\left({}^{3}\!/_{r}\right)(y_t - X_t'\theta)^2}{\sigma_t^2\Gamma\left({}^{1}\!/_{r}\right)}\right]^{\frac{r}{2}} \tag{16}$$

where the tail parameter $r > 0$ The GED is a normal distribution if $r = 2$, and fat-tailed if $r < 2$.

Model Selection Criteria

Akaike Information Criterion (AIC) due to (Akaike, 1974) is the most commonly used model selection criterion. The information criterion is computed as follows:

$$AIC(K) = -2\ln(L) + 2K \tag{17}$$

where K is the number of independently estimated parameters in the model, T is the number of observations; L is the maximized value of the Log - Likelihood for the estimated model and is defined by:

$$\left.\begin{aligned} L &= \prod_{i=0}^{n}\left(\frac{1}{2\pi\sigma_i^2}\right)^{1/2} exp\left[-\sum_{i=1}^{n}\frac{(y_i - f(x))^2}{2\sigma_i^2}\right] \\ \text{Log}\,L &= ln\left[\prod_{i=1}^{n}\left(\frac{1}{2\pi\sigma_i^2}\right)^{1/2}\right] - \frac{1}{2}\sum_{i=1}^{n}\frac{(y_i - f(x))^2}{\sigma_i^2} \end{aligned}\right\} \tag{18}$$

Thus given a set of estimated GARCH models for a given set of data, the preferred model is the one with the minimum information criteria and higher log likelihood value.

Volatility Half-Life

The half-life of volatility refers to the time taken by the volatility shock to cover half the distance back towards its mean volatility after a deviation from it (Engle and Bollerslev, 1986). For a stationary and mean reverting GARCH (1,1) model, the half-life of a volatility shock is given by the formula:

$$L_{Half} = 1 - \left\{ \frac{\log(2)}{\log(\alpha_1 + \beta_1)} \right\} \tag{19}$$

Results and discussion

Descriptive Statistics of Exchange Rate Returns

The descriptive statistics of the exchange rate log return series are summarized in Table 1.

Table 1: Descriptive Statistics of Exchange Rate Returns

Statistic	CFA	EURO	GBP	USD	WAUA	JPY
Mean	0.0327	0.0331	0.0248	0.0249	0.0288	0.0292
Sdt. Dev.	33.8592	19.1089	28.0757	5.4769	31.8820	17.1705
Skewness	1.3457	0.0421	0.0055	0.0300	-0.0313	-0.0136
Kurtosis	546.3312	1971.345	1002.632	1588.747	626.729	448.7506
Jarque-Bera	4.93E+08	6.47E+08	1.67E+08	4.20E+08	6.49E+08	3.31E+08
JB P-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
No of Obs.	4008	4008	4008	4008	4008	4008

From the descriptive statistics results presented in Table 1, it is observed that the daily means of the six exchange rates returns are all positive and very close to zero with high standard deviations indicating high level of dispersion from the daily average returns in the exchange market over the period under review. The WAUA and JYP exchange rates returns exhibit negative skewness while the CFA, EURO, GBP and USD returns exhibit positive skewness which are very close to zero except for the CFA. The positive or negative skewness indicate asymmetry in the return series. All the six exchange rates returns have high kurtosis values. The high values of kurtosis coefficients which deviate from normality suggest that big shocks of either signs are more likely to be present in the series and that the returns series exhibit leptokurtosis. The Jarque -Bera test which is performed at 1 percent significance level rejects the null hypothesis of zero skewness and kurtosis value of 3 in all cases. This confirms departure of the six exchange rate log returns from normality.

Unit Root Test Results

The unit root test results for the six exchange rates and their daily returns are presented in Table 2.

Table 2: ADF Unit Root Test Results

Variable	Option	ADF test statistic	P-value	1%	5%
cfa	Intercept only	-1.3303	0.7478	-3.4318	-2.8621
	Intercept & trend	-1.3470	0.7852	-3.9603	-3.4109
Δcfa	Intercept only	-25.9371**	0.0000	-3.4318	-2.8621
	Intercept & trend	-25.9339**	0.0000	-3.9603	-3.4109
$euro$	Intercept only	-0.6332	0.8641	-3.4318	-2.8621
	Intercept & trend	-1.3136	0.7952	-3.9603	-3.4109
$euro$	Intercept only	-26.0278**	0.0000	-3.4318	-2.8621
	Intercept & trend	-26.0245**	0.0000	-3.9603	-3.4109
gbp	Intercept only	-1.3255	0.7985	-3.4318	-2.8621
	Intercept & trend	-1.3581	0.8165	-3.9603	-3.4109
Δgbp	Intercept only	-25.1186**	0.0000	-3.4318	-2.8621
	Intercept & trend	-25.1158**	0.0000	-3.9603	-3.4109
usd	Intercept only	-0.6131	0.8654	-3.4318	-2.8621
	Intercept & trend	-2.0544	0.5706	-3.9603	-3.4109
Δusd	Intercept only	-30.3502**	0.0000	-3.4318	-2.8621
	Intercept & trend	-30.3701**	0.0000	-3.9603	-3.4109
$waua$	Intercept only	-0.3272	0.8218	-3.4318	-2.8621
	Intercept & trend	-1.3664	0.6584	-3.9603	-3.4109
$\Delta waua$	Intercept only	-24.5346**	0.0000	-3.4318	-2.8621
	Intercept & trend	-24.5326**	0.0000	-3.9603	-3.4109
jpy	Intercept only	-1.8982	0.8019	-3.4318	-2.8621
	Intercept & trend	-2.6145	0.5681	-3.9603	-3.4109
Δjpy	Intercept only	-27.6624**	0.0000	-3.4318	-2.8621
	Intercept & trend	-26.5027**	0.0000	-3.9603	-3.4109

Note: **denotes significance of the ADF test statistic at 1% & 5% levels. Δ denotes first difference operator.

The ADF unit root test results presented in Table 2 revealed that all the six exchange rates are non-stationary in level. This is indicated by their ADF test statistics being greater than their corresponding critical values at 1% and 5% significance levels with non-significant p-values. However, the returns (first differenced series) of the six exchange rates are stationary. This is indicated by their ADF test statistics being smaller than their corresponding critical values at 1% and 5% significance levels with significant p-values ($p < 0.05$).

The stationarity of the exchange rates returns means that no unit root is present in the foreign exchange rate returns.

Engle's LM Heteroskedasticity Test for ARCH Effects

The heteroskedasticity test results for ARCH effects of the six exchange rates return series is presented in Table 3.

Table 3: Heteroskedasticity Test Results

Variable	F-Statistic	P-value	nR^2	P-value
CFA	0.2873	0.0001	0.2876	0.0000
EURO	1.0752	0.0000	1.0752	0.0000
GBP	1.0037	0.0000	1.0039	0.0000
USD	3.8624	0.0000	3.8265	0.0000
WAUA	0.1109	0.0001	0.1113	0.0000
JPY	1.0965	0.0000	1.0969	0.0000

From the ARCH – LM test result presented in Table 3, both the F – statistics and nR^2 statistics in the entire six return series reject the null hypothesis of homoskedasticity. When the null hypothesis of no ARCH effect in the residuals of the series is rejected, it indicates the presence of ARCH effects in the residuals of returns. This means that the daily returns are non-constant and can only be modeled using Autoregressive Conditional Heteroskedastic Models.

Measures of Historical Volatility

The historical volatility measure (annualized volatility) for the six exchange rates return series together with the daily variance and daily standard deviation is presented in Table 4. The graphically representation using the daily standard deviation measure is reported in Figure 1.

Table 4: Historical Volatility Measure

Ranks	Daily Variance	Daily SD	Historical Volatility
CFA	1146.4454	33.8592	92.3716
WAUA	1016.4619	31.8820	89.6341
GBP	788.2449	28.0757	84.1135
EURO	365.1501	19.1089	69.3934
JPY	294.8261	17.1705	65.7797
USD	29.9964	5.4769	37.1508

The result of Table 4 shows the historical rankings of volatility for the six foreign exchange rates returns in Nigeria as measured by daily variance, daily standard deviation and annualized volatility methods. The result shows that CFA with daily standard deviation of 32.8592% and annualized volatility of 92.3716 ranked first as having the highest volatility (risk) among the six foreign exchange rates in Nigeria under review. Next to CFA is WAUA with daily standard deviation of 31.8820% and annualized volatility of 89.6341 and is second most volatile exchange rate. The third most volatile foreign exchange rate return series is the GBP with a daily standard deviation of 28.0757% and annualized volatility of 69.3934. The USD has the least daily variance measure, standard deviation and annualized volatility among the six exchange rates returns under review. Historical volatility is used as a criterion to study the risk associated with a financial asset. It is widely accepted as a practical measure of risk. Stocks with a high historical volatility are usually associated with a higher level of risk tolerance. From the result of historical volatility presented in Table 4, the USD is the only foreign exchange rate return associated with low level of risk. The historical ranking for the six exchange rates return series is presented graphically in Figure 1.

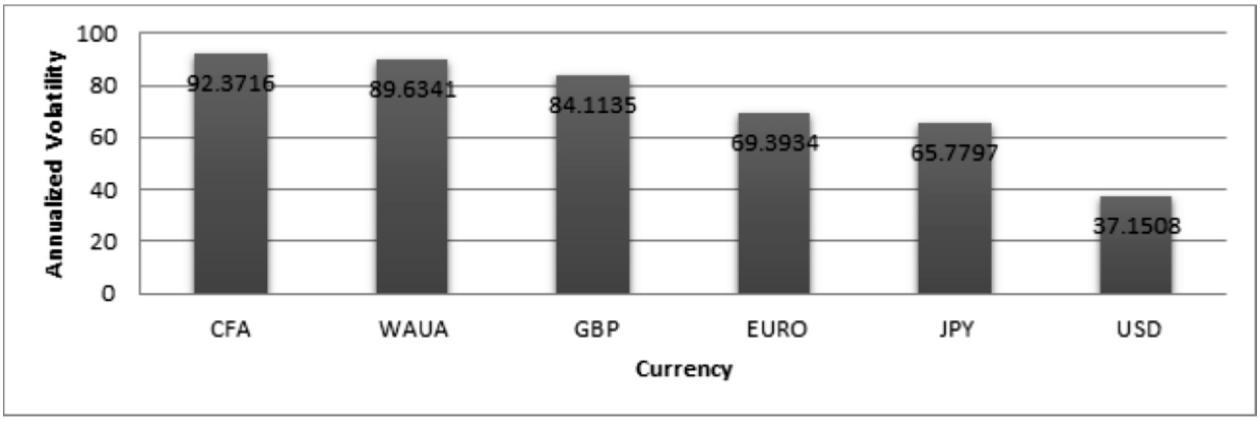

Figure 1: Plot of Historical Volatility Ranking of Some Selected Foreign Exchange Rates in Nigeria

The Choice of Innovation Density in Modeling Volatility

To select an innovation density (error distribution) that best fit a given exchange rate return series, we employ Akaike information criterion (AIC). The best fitted error distribution is given by the smallest information criterion. The result is presented in Table 5.

Table 5: Selection of Error Distribution using Akaike Information Criterion (AIC)

Distribution	CFA	EURO	GBP	USD	WAUA	JPY
			Symmetric GARCH (1,1) Model			
Gaussian	9.5326	8.2245	8.9299	6.0025	9.4925	8.2253
Student-t	**7.5159**	7.3387	6.6124	**3.4672**	8.4193	3.7702
GED	7.7201	**2.2416**	**5.0695**	4.1579	**8.1692**	**2.6629**
			Asymmetric EGARCH (1,1) Model			
Gaussian	9.1517	7.1120	8.7259	5.1069	9.0937	7.5186
Student-t	2.7362	**2.0322**	**1.9432**	4.8984	6.9099	2.4624
GED	**2.2658**	2.2978	2.2880	**3.7066**	**5.3413**	**2.1607**

The results of the choice of error distribution presented in Table 5 indicate that heavy tailed distributions are more suitable in modeling exchange rate volatility in Nigeria. Specifically, for symmetric models, student-t distribution is suitable for modeling Naira/CFA and Naira/USD exchange rates volatility while generalized error distribution (GED) is more suitable for modeling Naira/EURO, Naira/GBP, Naira/WAUA and Naira/JPY exchange rates volatility. For the asymmetric EGARCH models, student-t distribution is suitable for modeling Naira/EURO and Naira/GBP exchange rates volatility while generalized error distribution (GED) is more suitable for modeling Naira/CFA, Naira/USD, Naira/WAUA and Naira/JPY exchange rates volatility in Nigeria. This result is in accordance with the previous works of Thorlie *et al.* (2014), Omari *et al.* (2017) and Kuhe and Agaigbe (2018) which also found heavy-tailed error distributions more suitable in modeling exchange rate volatility.

Parameter Estimates of Symmetric GARCH Models

To investigate the symmetric properties of the six exchange rates return series, we employ the basic GARCH (1,1) model with different error distributions. The results are presented in Table 6.

Table 6: Parameter Estimates of Symmetric GARCH (1,1) Models with Fat-Tailed Distributions

Parameter	CFA	EURO	GBP	USD	WAUA	JPY
μ	0.0033	4.42E-08	0.6776	0.0309	0.0705	0.0258
ω	0.0868*	0.5923*	0.4249*	0.5341*	0.7894*	0.6397*
α_1	0.1769*	0.2394*	0.1548*	0.1202*	0.2083*	0.1408*
β_1	0.5896*	0.4557*	0.3299*	0.7328*	0.4257*	0.4957*
v	2.6679*	0.7174*	0.9749*	11.6731*	1.3590*	0.7522*
$\alpha_1+\beta_1$	**0.7665**	**0.6951**	**0.4847**	**0.8530**	**0.6340**	**0.5665**

Note: *denotes significant parameter, $\alpha_1+\beta_1$ denotes volatility shock persistence.

The parameter estimates of the symmetric GARCH (1,1) models presented in Table 6 shows that the coefficients of the ARCH terms (α_1) in all the six models are positive and statistically significant at 1% levels showing that news about previous volatilities have explanatory powers on current volatilities. The coefficients of GARCH terms (β_1) are also positive and statistically significant at 1% levels, showing that past volatilities of foreign exchange returns are significant and influence current volatilities. The sums of ARCH and GARCH coefficients which measures volatility shock persistence are less than one in all the models indicating that the models are stable and the conditional variances are stationary. This also implies that the volatilities are significantly quite persistence in Nigerian foreign exchange market. The results of the GARCH (1,1) models thus indicates that memories of shocks are remembered in Nigerian foreign Exchange Market. This finding agrees with the empirical findings of Kamal *et al.* (2012), David *et al.* (2016) and Kuhe and Agaigbe (2018) but contrary to the findings of Adeoye and Atanda (2011) and Musa and Abubakar (2014). Stationary and mean reverting returns offer good and long-term business opportunities for traders and investors.

Parameter Estimates of Asymmetric EGARCH Models

We employ the asymmetric EGARCH (1,1) models with different error distributions to investigate the asymmetric and leverage effect properties of the six exchange rates return series. The result is presented in Table 7.

Table 7: Parameter Estimates of Asymmetric EGARCH (1,1) Models with Fat-Tailed Distributions

Parameter	CFA	EURO	GBP	USD	WAUA	JPY
μ	0.0078	0.0189*	0.0097	0.0011	0.0049	0.0795
ω	0.3718*	0.3133*	0.3205*	0.2297*	0.0077*	0.7282*
α_1	0.1704*	0.1927*	0.1683*	0.1245*	0.1895*	0.2320*
γ	-0.0077*	-0.0759*	-0.1537*	-0.0153*	-0.0019*	-0.0155*
β_1	0.3184*	0.5236*	0.4849*	0.6388*	0.4763*	0.5471*
v	0.5857*	2.7108*	2.6492*	1.4309*	1.2634*	0.4865*
$\alpha_1+\beta_1$	**0.4888**	**0.7163**	**0.6532**	**0.7633**	**0.6658**	**0.7791**

Note: *denotes significant parameter, $\alpha_1+\beta_1$ denotes volatility shock persistence.

From the results of the asymmetric EGARCH models presented in Table 7, we observe that all the parameters of the estimated EGARCH (1,1) models in the variance equations are statistically significant at 5% significance levels and the mean reverting rates $\alpha_1+\beta_1$ are less than unity in all the exchange rates returns. This implies that the conditional variances of returns are stationary, stable, and predictable and mean revert to their long-run averages after deviating from it. The EGARCH (1,1) models also show that the leverage effect parameters are negative and statistically significant across the exchange rates returns suggesting that past negative shocks have greater impact on subsequent volatility than positive shocks of the same magnitudes. This result shows that asymmetry and leverage effects are indeed present in the Nigerian foreign exchange market. This result corroborates the findings of Abdalla (2012), Thorlie *et al.* (2014), David *et al.* (2016) and Kuhe and Agaigbe (2018) which also found asymmetry and leverage effect in exchange rate returns.

Volatility Mean Reversion and Half-Life

When a given series is stationary, it is mean reverting and volatility will eventually reverts to its long run average after a deviation from it (Engle and Bollerslev, 1986). We test for volatility mean reversion in the six exchange rates returns using GARCH family models. In a stationary GARCH (1,1) model, volatility mean reversion rate is given by the sum $\alpha_1+\beta_1$ which is generally close to unity for most financial data. The half-life of volatility represents the time taken by the volatility shock to cover half the distance back towards its mean volatility after a deviation from it. In our estimated models the mean reverting rates $|\alpha_1+\beta_1|$ which controls the speed of mean reversion are given in Table 8.

Table 8: Volatility Shock Persistence and Half-Lives

Variable	Model	Shock Persistence	Volatility Half-Life (in days)
CFA	GARCH (1,1)	0.7665	3
	EGARCH (1,1)	0.4888	1
EURO	GARCH (1,1)	0.6951	2
	EGARCH (1,1)	0.7163	2
GBP	GARCH (1,1)	0.4847	1
	EGARCH (1,1)	0.6532	2
USD	GARCH (1,1)	0.8530	4
	EGARCH (1,1)	0.7633	3
WAUA	GARCH (1,1)	0.6340	2
	EGARCH (1,1)	0.6658	2
JPY	GARCH (1,1)	0.5665	1
	EGARCH (1,1)	0.7791	3

The half life of a volatility shock measures the average time it takes for $|\varepsilon_t^2 - \hat{\sigma}^2|$ to decrease by one half. In the estimated symmetric and asymmetric GARCH models, the volatility half-lives vary from one to four days depending on the model, return series and the error distribution used. This result indicates how short and frequent it takes for the foreign exchange rates to come back to normal (average) level in Nigerian foreign exchange market. This result also indicates that no matter how high or low the six exchange rates shall move in the Nigerian exchange market, they shall eventually revert to a long-run average level. This result is similar to the findings of Kuhe and Agaigbe (2018).

Model Diagnostic Checks

We conduct model adequacy checks using Engle's LM heteroscedasticity test and Ljung-Box Q-statistics of the standardized residuals of the estimated models. The results are presented in Table 9.

Table 9: Post Estimation Heteroskedasticity and Serial CorrelationTests Results

Variable	Model	F-Statistic	P-value	Q-Stat.	P-value
CFA	GARCH (1,1)	0.1266	0.7220	0.1268	0.7223
	EGARCH (1,1)	0.0030	0.9565	0.0030	0.9565
EURO	GARCH (1,1)	0.0288	0.8653	0.0288	0.8652
	EGARCH (1,1)	0.0003	0.9871	0.0003	0.9872
GBP	GARCH (1,1)	0.0921	0.7616	0.0922	0.7614
	EGARCH (1,1)	0.0009	0.9748	0.0020	0.9997
USD	GARCH (1,1)	0.0014	0.9701	0.0014	0.9703
	EGARCH (1,1)	0.0007	0.9789	15.200	0.1257
WAUA	GARCH (1,1)	0.2025	0.6528	0.2055	0.9026
	EGARCH (1,1)	0.0026	0.9596	0.0051	0.9975
JPY	GARCH (1,1)	0.0379	0.8457	0.0379	0.8462
	EGARCH (1,1)	0.0052	0.9426	0.0052	0.9426

From the result of the heteroskedasticity test for ARCH effect presented in Table 9, we fail to reject the null hypothesis of no ARCH effect in the residuals of returns as indicated by the non-significant p-values of the F-statistics. This is because the estimated GARCH models have adequately captured all the ARCH effects and none is remaining in the residuals of returns. The Ljung-Box Q-statistics of the standardized residuals of the estimated GARCH models which are highly statistically insignificant indicate the absence of autocorrelation in the standardized residuals of the estimated models. This shows that the estimated GARCH type models are good fits to the exchange rate return data.

Conclusion

Volatility is used as a criterion to study the risk associated with any financial asset. It is widely accepted as a practical measure of financial risk. Stocks with a high volatility are usually associated with a higher level of risk tolerance whereas stationary and mean reverting stocks are associated with low level of risk tolerance. This study investigated the volatility behaviour of exchange rates returns of Naira against CFA, Euro, Great British Pounds, US Dollar, West African Unit of Account (WAUA) and Japanese Yen in Nigeria using historical volatility approach as well as symmetric and asymmetric GARCH models with non-Gaussian errors. The study utilized daily quotations of these exchange rates from 12/11/2001 to 04/13/2018. The study employed the popular Dickey-Fuller unit root test to investigate the order of integration of the series, Engle's LM test to detect the presence of ARCH effects in the residuals of exchange rate returns. Historical (annualized) volatility approach as well as symmetric GARCH (1,1) and asymmetric EGARCH (1,1) models were used to model the exchange rates return series. Results showed that the daily exchange rates are non-stationary while their returns are stationary. The ARCH test result showed the presence of ARCH effects in the residuals of exchange rates returns. The historical measure of volatility showed that CFA and USD have the highest and least annualized volatilities (market risk) respectively among the six exchange rates returns. The symmetric GARCH (1,1) model showed volatility clustering with evidence of shock persistence in the six exchange rates return series. The asymmetric EGARCH (1,1) model found evidence of asymmetry and leverage effects in the Nigerian foreign exchange market indicating that negative shocks (bad news) generate more volatility than positive shocks (good news) of similar magnitudes. All the estimated models were found to be stationary and mean reverting indicating the predictability and stability of the conditional variances of the foreign exchange rates returns. This result suggests that no matter how high or low the foreign exchange rates shall move in the exchange market, they shall eventually return to their long-run averages. Stationary and mean reverting stocks provide good and long term investment opportunities for investors.

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