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A Power Transformation Ratio-type Estimator of Finite Population Mean in Stratified Random Sampling

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Abstract
In this paper, a ratio-type estimator of finite population mean in stratified random sampling based on the Srivastava(1967) estimator have been theoretically and empirically studied. Theoretically, the bias and variance for the separate and combined estimators were obtained and compared it with the bias and variance of the traditional separate and combined ratio estimators. Other criteria used to determine the most efficient estimators are: standard Error (SE), Coefficient of Variation (CV) and Percentage Relative Efficiency (PRE). Comparison of the estimators empirically, shows that the modified estimators are more efficient than the traditional separate and combined ratio estimators in all conditions. In addition, varying values of α indicates that even if α deviates from its exact optimum values the modified estimators $\bar{y}_{\alpha s}$ and $\bar{y}_{\alpha c}$ will be better estimators than the usual estimators $\bar{y}, \bar{y}_{RS}, \bar{y}_{RC}$ .Based on such findings, we suggest the use of the proposed estimator in stratified random sampling as better estimators over the traditional ratio estimators in stratified random sampling for practical situations.

Keywords: Power transformation, Population mean, Stratified random sampling, Auxiliary variable.

Introduction
Estimation of population characteristics is crucial in drawing meaningful conclusions on data from surveys. One such estimator that has been found useful due to its versatility and efficiency is the ratio estimator. Of cause, they are useful when estimating population parameters that utilize information on the auxiliary variable in order to enhance efficiency of estimation and when the regression line passes through the origin. Sometimes, where possible, stratification is performed as it lowers the variance of the estimator thereby boosting efficiency. This strategy can significantly better the estimate accuracy, resulting in an estimate that conforms to the population values.

Several studies have been done on the use of auxiliary information to enhance the efficiency of an estimator in stratified random sampling.

Singh and Ahmed (2014) developed some ratio-type estimators in stratified random sampling for estimating population mean by using information on auxiliary attributes. They found that the proposed estimators are more efficient and less bias than the traditional combined ratio estimator. Tailor and Lone (2014) considered the separate ratio-type estimators for population mean with their properties for population mean using known parameters of auxiliary variate. It was shown that the proposed estimators are more efficient than the unbiased estimators in stratified random sampling and the usual separate ratio estimators under certain obtained conditions. Verma *et al.* (2015) suggested some generalized classes of modified ratio, regression-cum-ratio and exponential ratio type estimators for finite population mean of the study variable utilizing the information on two auxiliary variables in stratified random sampling. The empirical study showed that the suggested estimators were better. Subzar *et al.* (2018) examined the estimation of finite population mean in stratified random sampling using non-conventional measures of dispersion. They concluded that their estimators performed better than the conventional estimators as their mean square error and bias were lower than the conventional estimators. Sharma and Kumar (2020) studied the properties of ratio type estimator in stratified random sampling using multi-auxiliary variables. Through simulation study, they concluded that the proposed estimator performed better than the estimators in their study. Recently, Khalid *et al.* (2022) proposed a dual ratio cum product type of the exponential estimator for the population mean under the stratified random sampling design. Their results showed that the dual operation enormously improved the efficiency of the estimators. Bhushan *et al.* (2023) proposed an efficient class of estimators in stratified random sampling with an application to real data. Their results suggested that the class of estimators proposed is more effective than the other available estimators. Singh *et al.* (2023) developed a class of estimators for finite population mean using auxiliary attribute in stratified random sampling. Their results showed that the suggested classes of estimators perform better than other existing methods, with lowest mean square error under optimal conditions. Shahzad *et al.* (2019) suggested a family of ratio estimators in stratified random sampling utilizing auxiliary attribute along the nonresponse issue. The proposed class of estimators was more efficient than the customary ratio, regression, Bal and Tuteja (1991), Diana (1993), Kadila and Cingi (2003), Shabbir and Gupta (2011), Koyuncu (2016).

The use of power transformation technique stabilizes variance and also improves the validity of an estimator. There is paucity literature on ratio-type estimators using power transformation applied in stratified random sampling. Singh *et al.* (2008) suggested two modified estimators of population mean using power transformation in stratified random sampling. Mehta and Sharma (2023) proposed generalized power transformation estimators in stratified ranked set of sampling using auxiliary. Ahmad and Singh (2021) suggested a separate exponential ratio- type estimator of finite population mean under Power Transformation. Onyeka *et al.* (2015) proposed separate-type estimators for estimating population ratio in post-stratified sampling using variable transformation. Swain *et al.* (2022) constructed a family of almost unbiased transformed ratio estimators and further extended it to stratified random sampling with transformed ratio estimator in each stratum. Grover and Kaur (2014) proposed a general class of ratio-type exponential estimators of population mean under linear transformation of auxiliary variable. The aim of this study is to implement the Srivastava (1967) power transformation estimator to estimate the population means in stratified random sampling without replacement considering separate and combined estimation techniques.

Preliminaries

The following are the notations used under stratified random sampling L ; Number of strata in the population. N_h ; Number of units in the stratum h, h=1,2,…,L

$\bar{Y}_h = \frac{1}{N_h}\sum_{h=1}^{N_h} y_{hi}$; h^{th} stratum population mean of the study variable., $\bar{X}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$; h^{th} stratum population mean of the auxiliary variable.

$$\bar{Y} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi} = \frac{1}{N_h}\sum_{i=1}^{L} N_h\bar{Y}_h = \sum_{h=1}^{L} W_h\bar{Y}_h = \bar{Y}_{st} ;$$

Population mean of the stratified sampling for variable Y. $W_h = \frac{N_h}{N}$; Stratum weight of h^{th} stratum.

$\overline{X} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} x_{hi} = \frac{1}{N_h}\sum_{i=1}^{L} N_h \overline{X}_h = \sum_{h=1}^{L} W_h \overline{X}_h = \overline{X}_{st} =$ Population mean of the stratified sampling for auxiliary variable X.

$\overline{y}_h = \frac{1}{n_h}\sum_{h=1}^{n_h} y_{hi}$ = Sample mean of the study variable Y in the h^{th} stratum, $\overline{x}_h = \frac{1}{n_h}\sum_{h=1}^{n_h} x_{hi}$ = Sample mean of the study variable X in the h^{th} stratum. $R_h = \frac{\overline{Y}_h}{\overline{X}_h}$ = Ratio of the population means in the stratum.

$S_{Yh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(Y_{hi} - \overline{Y}_h)^2$ = Population mean square error of the study variable for the h^{th} stratum. $S_{Xh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(X_{hi} - \overline{X}_h)^2$; Population mean square error of the study variable for the h^{th} stratum. $S_{Xh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(X_{hi} - \overline{X}_h)^2$; Population mean square error of the auxiliary variable for the h^{th} stratum.

$S_{XYh} = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(X_{hi} - \overline{X}_h)(Y_{hi} - \overline{Y}_h) =$ Population covariance between auxiliary variable and study variable for h^{th} stratum. $C_{Yh} = \frac{S_{Yh}}{\overline{Y}_h}$; Population coefficient of variation of the study variable for h^{th} stratum. $C_{Xh} = \frac{S_{Xh}}{\overline{X}_h}$; Population coefficient of variation of the auxiliary variable for h^{th} stratum, $\rho_h = \frac{S_{XY}}{S_X S_Y}$ = Population correlation coefficient between the auxiliary variable and study variable for the h^{th} stratum. $f_h = \frac{n_h}{N_h}$; The h^{th} stratum sampling fraction.

$\overline{y}_{st} = \sum_{h=1}^{L} W_h \overline{y}_h$ = Mean of stratified sampling of the study variable under SRS, $\overline{x}_{st} = \sum_{h=1}^{L} W_h \overline{x}_h$; Mean of stratified sampling of the auxiliary variable under SRS.

Consider the transformations $\varepsilon_0 = \frac{\overline{y} - \overline{Y}}{\overline{Y}}$, $\varepsilon_1 = \frac{\overline{x} - \overline{X}}{\overline{X}}$, where $\overline{y} = \overline{Y}(1+\varepsilon_0)$, $\overline{x} = \overline{X}(1+\varepsilon_1)$.

The following assumptions are made

$E(\varepsilon_0) = E(\varepsilon_1) = 0$, $E(\varepsilon_0^2) = \frac{1-f}{n}C_y^2$,

$E(\varepsilon_1^2) = \frac{1-f}{n}C_x^2$, $E(\varepsilon_0\varepsilon_1) = \frac{1-f}{n}\rho C_x C_y$

ESTIMATORS IN STRATIFIED RANDOM SAMPLING

Conventional Separate Ratio Estimator in Stratified Random Sampling

Let $\overline{y}_h$ and $\overline{x}_h$, h=1, 2… L is an unbiased for the population means $\overline{Y}_h$ and $\overline{X}_h$, h^{th} in the stratum for the study and auxiliary variables respectively, then the separate ratio estimator is given as

$$\overline{y}_{RS} = \sum_{h=1}^{L} W_h \overline{y}_h \left(\frac{\overline{X}_h}{\overline{x}_h}\right) \tag{1}$$

The bias and variance of the separate ratio estimator $\overline{y}_{RS}$ is given as

$$B(\overline{y}_{RS}) = \sum_{h=1}^{L} W_h \left(\frac{1-f_h}{n_h}\right)\frac{1}{\overline{X}_h}\left[R_h S_{hx}^2 - S_{hxy}\right] \tag{2}$$

and

$$V(\overline{y}_{RS}) = \sum_{h=1}^{L} W_h^2 \left(\frac{1-f_h}{n_h}\right)\left[S_{hy}^2 + R_h^2 S_{hx}^2 - 2R_h S_{hxy}\right] \tag{3}$$

Conventional Combined Ratio Estimator in Stratified Random Sampling

The combined ratio estimator $\overline{y}_{RC}$ of population mean $\overline{Y}$ is defined as.

$$\overline{y}_{RC} = \overline{y}_{st}\left(\frac{\overline{X}}{\overline{x}_{st}}\right) \tag{4}$$

The bias and variance of the combined estimator $\overline{y}_{RC}$ is given by

$$B(\overline{y}_{RC}) = \sum_{h=1}^{L} W_h^2 \left(\frac{1-f_h}{n_h}\right)\frac{1}{\overline{X}}\left[R S_{hx}^2 - S_{hxy}\right] \tag{5}$$

and

$$V(\bar{y}_{RC}) = \sum_{h=1}^{L} W_h^2 \left(\frac{1-f_h}{n_h}\right)\left[S_{hy}^2 + R^2 S_{hx}^2 - 2RS_{hxy}\right] \quad (6)$$

MODIFIED RATIO-TYPE ESTIMATORS

Proposition I: Motivated by Srivastava (1967) we propose a modified separate and combined ratio-type estimator using auxiliary variable in stratified sampling as

$$\bar{y}_{\alpha s} = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{X}_h}{\bar{x}_h}\right)^{\alpha},\ \alpha > 0 \quad (7)$$

Where, α is a real constant to be suitably determined such that the variance of the estimator is minimum.

Theorem I: The bias of the proposed estimator is given as (8).

$$B(\bar{y}_{\alpha s}) = \sum_{h=1}^{L} W_h \left(\frac{1-f_h}{n_h}\right)\frac{1}{\bar{X}_h}\left[\frac{\alpha(\alpha+1)}{2} R_h S_{hx}^2 - \alpha S_{hxy}\right] \quad (8)$$

Proof:

Suppose that $\varepsilon_{0h} = \dfrac{\bar{y}_h - \bar{Y}_h}{\bar{Y}_h}, \Rightarrow \bar{y}_h = \bar{Y}_h(1+\varepsilon_0)$,

$$\varepsilon_{ih} = \frac{\bar{x}_h - \bar{X}_h}{\bar{X}_h}, \Rightarrow \bar{x}_h = \bar{X}_h(1+\varepsilon_{ih})$$

Thus, $E(\varepsilon_{0h}) = E(\varepsilon_{1h}) = 0$,

$$E(\varepsilon_{0h}^2) = \left(\frac{1-f_h}{n_h}\right)\frac{S_{hy}^2}{\bar{Y}_h^2}, E(\varepsilon_{ih}^2) = \left(\frac{1-f_h}{n_h}\right)\frac{S_{hx}^2}{\bar{X}_h^2},$$

$$E(\varepsilon_{0h}\varepsilon_{ih}) = \left(\frac{1-f_h}{n_h}\right)\frac{S_{hxy}}{\bar{X}_h \bar{Y}_h}$$

Where: $f_h = \dfrac{n_h}{N_h}$, $S_{hy}^2 = \dfrac{1}{n_h - 1}\sum_{i=1}^{n_h}(y_i - \bar{y}_h)^2$,

$$S_{hx}^2 = \frac{1}{n_h - 1}\sum_{i=1}^{n_h}(x_i - \bar{x}_h)^2$$

Expressing (7) in terms of ε_{oh} and ε_{ih} the modified estimator becomes

$$\bar{y}_{\alpha s} = \sum_{h=1}^{L} W_h \bar{Y}_h (1+\varepsilon_{oh})\left(\frac{\bar{X}_h}{\bar{X}_h(1+\varepsilon_{ih})}\right)^{\alpha}$$

$$= \sum_{h=1}^{L} W_h \bar{Y}_h (1+\varepsilon_{oh})(1+\varepsilon_{ih})^{-\alpha} \quad (9)$$

Using Taylor's series expansion and ignoring powers higher than two, (9) becomes

$$\bar{y}_{\alpha s} = \sum_{h=1}^{L} W_h \bar{Y}_h (1+\varepsilon_{oh})\left[1 - \alpha\varepsilon_{ih} + \frac{\alpha(\alpha+1)}{2}\varepsilon_{ih}^2 - \cdots\right]$$

Expanding the brackets and ignoring powers higher than two, the estimator becomes

$$= \sum_{h=1}^{L} W_h \bar{Y}_h \left[1 + \varepsilon_{oh} - \alpha\varepsilon_{ih} + \frac{\alpha(\alpha+1)}{2}\varepsilon_{ih}^2 - \alpha\varepsilon_{0h}\varepsilon_{ih}\right]$$

$$= \sum_{h=1}^{L} W_h \bar{Y}_h + \sum_{h=1}^{L} W_h \bar{Y}_h \left[\varepsilon_{oh} - \alpha\varepsilon_{ih} + \frac{\alpha(\alpha+1)}{2}\varepsilon_{ih}^2 - \alpha\varepsilon_{0h}\varepsilon_{ih}\right]$$

$$= \bar{Y} + \sum_{h=1}^{L} W_h \bar{Y}_h \left[\varepsilon_{oh} - \alpha\varepsilon_{ih} + \frac{\alpha(\alpha+1)}{2}\varepsilon_{ih}^2 - \alpha\varepsilon_{0h}\varepsilon_{ih}\right]$$

$$\bar{y}_{\alpha s} - \bar{Y} = \sum_{h=1}^{L} W_h \bar{Y}_h \left[\varepsilon_{oh} - \alpha\varepsilon_{ih} + \frac{\alpha(\alpha+1)}{2}\varepsilon_{ih}^2 - \alpha\varepsilon_{0h}\varepsilon_{ih}\right] \quad (10)$$

Taking expectation on both sides of (10) the bias is gotten as:

$$E(\bar{y}_{\alpha s} - \bar{Y}) = \sum_{h=1}^{L} W_h \bar{Y}_h E\left[\varepsilon_{oh} - \alpha\varepsilon_{ih} + \frac{\alpha(\alpha+1)}{2}\varepsilon_{ih}^2 - \alpha\varepsilon_{0h}\varepsilon_{ih}\right]$$

$$= \sum_{h=1}^{L} W_h \bar{Y}_h \left[E(\varepsilon_{oh}) - \alpha E(\varepsilon_{ih}) + \frac{\alpha(\alpha+1)}{2}E(\varepsilon_{ih}^2) - \alpha E(\varepsilon_{0h}\varepsilon_{ih})\right]$$

$$= \sum_{h=1}^{L} W_h \bar{Y}_h \left(\frac{1-f_h}{n_h}\right)\left[\frac{\alpha(\alpha+1)}{2}\frac{S_{hx}^2}{\bar{X}_h^2} - \alpha\frac{S_{hxy}}{\bar{X}_h \bar{Y}_h}\right]$$

Therefore the bias of the proposed separate estimator is

$$B(\bar{y}_{\alpha s}) = E(\bar{y}_{\alpha s} - \bar{Y}) = \sum_{h=1}^{L} W_h \left(\frac{1-f_h}{n_h}\right)\frac{1}{\bar{X}_h}\left[\frac{\alpha(\alpha+1)}{2} R_h S_{hx}^2 - \alpha S_{hxy}\right] \quad (11)$$

Theorem II: The variance of the separate modified ratio type estimator of the finite population mean under stratified random sampling is given by:

$$V(\bar{y}_{\alpha s}) = \sum_{h=1}^{L} W_h^2 \left(\frac{1-f_h}{n_h}\right)(S_{hy}^2 + \alpha^2 R_h^2 S_{hx}^2 - 2\alpha R_h S_{hxy}) \quad (12)$$

Proof:

$$V(\bar{y}_{\alpha s}) = E\left[\bar{y}_{\alpha s} - E(\bar{y}_{\alpha s})\right]^2 = E(\bar{y}_{\alpha s})^2 - \left[E(\bar{y}_{\alpha s})\right]^2$$

$$= \sum_{h-1}^{L} W_h^2 E\left[\bar{Y}_h(1+\varepsilon_{0h})\left(\frac{\bar{X}_h}{\bar{X}_h(1+\varepsilon_{1h})}\right)^{\alpha}\right]^2 - \bar{Y}^2$$

$$= \sum_{h-1}^{L} W_h^2 E\left[\bar{Y}_h(1+\varepsilon_{0h})\left(\frac{1}{(1+\varepsilon_{1h})}\right)^{\alpha}\right]^2 - \bar{Y}^2$$

$$=\sum_{h-1}^{L} W_h^2 E\left[\bar{Y}_h(1+\varepsilon_{0h})(1+\varepsilon_{ih})^{-\alpha}\right]^2-\bar{Y}^2$$

$$=\sum_{h-1}^{L} W_h^2 E\left[\bar{Y}_h(1+\varepsilon_{0h})(1-\alpha\varepsilon_{ih}+\frac{\alpha(\alpha+1)}{2}\varepsilon_{ih}^2)\right]^2-\bar{Y}^2$$

$$=\sum_{h=1}^{L} W_h^2\bar{Y}_h^2 E\left[1+\varepsilon_{oh}-\alpha\varepsilon_{ih}+\frac{\alpha(\alpha+1)}{2}\varepsilon_{ih}^2-\alpha\varepsilon_{0h}\varepsilon_{ih}\right]^2-\bar{Y}^2$$

$$=\sum_{h=1}^{L} W_h^2\bar{Y}_h^2 E\left[1+\varepsilon_{0h}^2-\alpha\varepsilon_{oh}\varepsilon_{ih}-\alpha\varepsilon_{0h}\varepsilon_{ih}+\alpha^2\varepsilon_{ih}^2\right]-\bar{Y}^2$$

$$=\sum_{h=1}^{L} W_h^2\bar{Y}_h^2+\sum_{h=1}^{L} W_h^2\bar{Y}_h^2 E\left[\varepsilon_{0h}^2+\alpha^2\varepsilon_{ih}^2-2\alpha\varepsilon_{oh}\varepsilon_{ih}\right]-\bar{Y}^2$$

$$=\bar{Y}^2+\sum_{h=1}^{L} W_h^2\bar{Y}_h^2 E\left[\varepsilon_{0h}^2+\alpha^2\varepsilon_{ih}^2-2\alpha\varepsilon_{oh}\varepsilon_{ih}\right]-\bar{Y}^2$$

$$=\sum_{h=1}^{L} W_h^2\bar{Y}_h^2\left[E(\varepsilon_{0h}^2)+\alpha^2E(\varepsilon_{ih}^2)-2\alpha E(\varepsilon_{oh}\varepsilon_{ih})\right]$$

$$V(\bar{y}_{\alpha s})=\sum_{h=1}^{L} W_h^2\left(\frac{1-f_h}{n_h}\right)\left[S_{hy}^2+\alpha^2R_h^2S_{hx}^2-2\alpha R_hS_{hxy}\right] \quad (13)$$

To obtain the optimum value of α that optimizes the $V(\bar{y}_{\alpha s})$ take the partial derivative of (13) and simplify to obtain the value of as

$$\frac{\partial V(\bar{y}_{\alpha s})}{\partial\alpha}=2\alpha R_h^2S_{hx}^2-2R_hS_{hxy}=0$$

$$2\alpha R_h^2S_{hx}^2-2R_hS_{hxy}=0$$

$$2\alpha R_h^2S_{hx}^2=2R_hS_{hxy}$$

$$\alpha R_h^2S_x^2=R_hS_{hxy}$$

$$\alpha=\frac{S_{hxy}}{R_hS_{hx}^2} \quad (14)$$

Proposition II: The modified combined ratio-type estimator of the under stratified random sampling survey is given as

$$\bar{y}_{\alpha c}=\bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)^{\alpha}, \quad \alpha>0 \quad (15)$$

Where, $\bar{y}_{st}=\sum_{h=1}^{L}W_h\bar{y}_h$, $\bar{x}_{st}=\sum_{h=1}^{L}W_h\bar{x}_h$, L is the number of stratum, $W_h=\frac{N_h}{N}$ is the stratum weight, N is the number of units in the population, N_h is the number of units in stratum h, $\bar{y}_h$ is the sample mean of the variable of interest in stratum h and $\bar{x}_h$ is the sample mean of the auxiliary variable in stratum h.

Theorem III: The bias of the combined estimator under stratified random sampling is given as

$$B(\bar{y}_{\alpha c})=\sum_{h=1}^{L}W_h^2\left(\frac{1-f_h}{n_h}\right)\frac{1}{\bar{X}}\left[\frac{\alpha(\alpha+1)}{2}RS_{hx}^2-\alpha S_{hxy}\right] \quad (16)$$

Proof:

Supposed $\varepsilon_0=\frac{\bar{y}_{st}-\bar{Y}}{\bar{Y}},\Rightarrow\bar{y}_{st}=\bar{Y}(1+\varepsilon_0)$,

$$\varepsilon_1=\frac{\bar{x}_{st}-\bar{X}}{\bar{X}},\Rightarrow\bar{x}_{st}=\bar{X}(1+\varepsilon_1)$$

$$E(\varepsilon_0)=E(\varepsilon_1)=0$$

Assuming that the strata are independent, then

$$E(\varepsilon_0)=\sum_{h=1}^{L}W_h^2\left(\frac{1-f_h}{n_h}\right)\frac{S_{hy}^2}{\bar{Y}^2},$$

$$E(\varepsilon_1)=\sum_{h=1}^{L}W_h^2\left(\frac{1-f_h}{n_h}\right)\frac{S_{hx}^2}{\bar{X}^2},$$

$$E(\varepsilon_0\varepsilon_1)=\sum_{h=1}^{L}W_h^2\left(\frac{1-f_h}{n_h}\right)\frac{S_{hxy}^2}{\bar{X}\bar{Y}}$$

Substituting the value $\bar{y}_{st}$ and $\bar{x}_{st}$ (18) then:

$$\bar{y}_{\alpha c}=\bar{Y}(1+\varepsilon_0)\left(\frac{\bar{X}}{\bar{X}(1+\varepsilon_1)}\right)^{\alpha}$$

$$=\bar{Y}(1+\varepsilon_0)\left(\frac{1}{(1+\varepsilon_1)}\right)^{\alpha}$$

$$=\bar{Y}(1+\varepsilon_0)(1+\varepsilon_1)^{-\alpha}$$

Expanding (17) using Taylor's series and ignoring powers higher than two (17) becomes (17)

$$\bar{y}_{\alpha c}=\bar{Y}(1+\varepsilon_0)(1-\alpha\varepsilon_1+\frac{\alpha(\alpha+1)}{2}\varepsilon_1^2-\cdots)$$

$$=\bar{Y}(1+\varepsilon_0-\alpha\varepsilon_1+\frac{\alpha(\alpha+1)}{2}\varepsilon_1^2-\alpha\varepsilon_0\varepsilon_1)$$

$$=\bar{Y}+\bar{Y}(\varepsilon_0-\alpha\varepsilon_1+\frac{\alpha(\alpha+1)}{2}\varepsilon_1^2-\alpha\varepsilon_0\varepsilon_1)$$

$$\bar{y}_{\alpha c}-\bar{Y}=\bar{Y}(\varepsilon_0-\alpha\varepsilon_1+\frac{\alpha(\alpha+1)}{2}\varepsilon_i^2-\alpha\varepsilon_0\varepsilon_1) \quad (18)$$

Taking expectation on both sides of (18)
the bias is obtained as

$$E(\bar{y}_{st}-\bar{Y})=E\left[\bar{Y}(\varepsilon_0-\alpha\varepsilon_1+\frac{\alpha(\alpha+1)}{2}\varepsilon_1^2-\alpha\varepsilon_0\varepsilon_1\right]$$

$$E(\bar{y}_{st})-\bar{Y}=\bar{Y}E\left[(\varepsilon_0-\alpha\varepsilon_1+\frac{\alpha(\alpha+1)}{2}\varepsilon_1^2-\alpha\varepsilon_0\varepsilon_1\right]$$

$$= \bar{Y}\left[E(\varepsilon_0) - \alpha E(\varepsilon_1) + \frac{\alpha(\alpha+1)}{2}E(\varepsilon_1^2) - \alpha E(\varepsilon_0\varepsilon_1)\right]$$

$$B(\bar{y}_{\alpha c}) = E(\bar{y}_{st}) - \bar{Y} = \sum_{h=1}^{L} W_h^2\left(\frac{1-f_h}{n_h}\right)\frac{1}{\bar{X}}\left[\frac{\alpha(\alpha+1)}{2}RS_{hx}^2 - \alpha S_{hxy}\right] \quad (19)$$

Theorem IV: The Variance of the combined estimator under stratified random sampling is given as

$$V(\bar{y}_{\alpha c}) = \sum_{h=1}^{L} W_h^2\left(\frac{1-f_h}{n_h}\right)\left[S_{hy}^2 + \alpha^2R^2S_{hx}^2 - 2\alpha RS_{hxy}\right] \quad (20)$$

Proof:

$$V(\bar{y}_{\alpha c}) = E[\bar{y}_{\alpha c} - E(\bar{y}_{\alpha c})]^2 = E(\bar{y}_{\alpha c})^2 - [E(\bar{y}_{\alpha c})]^2$$

$$= E\left[\bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)^{\alpha}\right]^2 - \bar{Y}^2$$

$$= E\left[\bar{Y}(1+\varepsilon_0)\left(\frac{\bar{X}}{\bar{X}(1+\varepsilon_1)}\right)^{\alpha}\right]^2 - \bar{Y}^2$$

$$= E\left[\bar{Y}(1+\varepsilon_0)(1+\varepsilon_1)^{-\alpha}\right]^2 - \bar{Y}^2$$

$$= E\left[\bar{Y}(1+\varepsilon_0)(1-\alpha\varepsilon_1 + \frac{\alpha(\alpha+1)}{2}\varepsilon_1^2 - \cdots)\right]^2 - \bar{Y}^2$$

$$= E\left[\bar{Y}(1+\varepsilon_0 - \alpha\varepsilon_1 + \frac{\alpha(\alpha+1)}{2}\varepsilon_1^2 - \alpha\varepsilon_0\varepsilon_1)\right]^2 - \bar{Y}^2$$

$$= E\left[\bar{Y}^2(1+\varepsilon_0 - \alpha\varepsilon_1 + \frac{\alpha(\alpha+1)}{2}\varepsilon_1^2 - \alpha\varepsilon_0\varepsilon_1)^2\right] - \bar{Y}^2$$

$$= E\left[\bar{Y}^2(1+\varepsilon_0^2 + \alpha^2\varepsilon_1^2 - 2\alpha\varepsilon_0\varepsilon_1)\right] - \bar{Y}^2$$

$$= E\left[\bar{Y}^2 + \bar{Y}^2(\varepsilon_0^2 + \alpha^2\varepsilon_1^2 - 2\alpha\varepsilon_0\varepsilon_1)\right] - \bar{Y}^2$$

$$= \bar{Y}^2 + \bar{Y}^2E(\varepsilon_0^2 + \alpha^2\varepsilon_1^2 - 2\alpha\varepsilon_0\varepsilon_1) - \bar{Y}^2$$

$$= \bar{Y}^2\left[E(\varepsilon_0^2) + \alpha^2E(\varepsilon_1^2) - 2\alpha E(\varepsilon_0\varepsilon_1)\right]$$

$$V(\bar{y}_{\alpha c}) = \sum_{h=1}^{L} W_h^2\left(\frac{1-f_h}{n_h}\right)\left[S_{hy}^2 + \alpha^2R^2S_{hx}^2 - 2\alpha RS_{hxy}\right] \quad (21)$$

To obtain the Optimum value of α that optimizes the $V(\bar{y}_{\alpha c})$, take the partial derivative of (21) and simplify to obtain the value of as

$$\frac{\partial V(\bar{y}_{\alpha s})}{\partial \alpha} = 2\alpha R^2S_{hx}^2 - 2RS_{hxy} = 0$$

$$2\alpha R^2S_{hx}^2 - 2RS_{hxy} = 0$$

$$2\alpha R^2S_{hx}^2 = 2RS_{hxy}$$

$$\alpha R^2S_x^2 = RS_{hxy}$$

$$\alpha = \frac{S_{hxy}}{RS_{hx}^2} \quad (22$$

THEORETICAL COMPARISON

To compare the efficiency of the proposed stratified ratio type estimator with that of the existing estimators under Stratified random sampling, we first of all compare their separate estimators and combined estimators with the variances of the existing estimators.

Comparing (3) and (12) then:

$$V(\bar{y}_{RS}) > V(\bar{y}_{\alpha s}) \Rightarrow V(\bar{y}_{RS}) - V(\bar{y}_{\alpha s}) > 0 \quad (23)$$

$V(\bar{y}_{\alpha s})$ is more efficient otherwise $V(\bar{y}_{RS})$ is more efficient.

Comparing (6) and (20) then:

$$V(\bar{y}_{RC}) > V(\bar{y}_{\alpha c}) \Rightarrow V(\bar{y}_{RC}) - V(\bar{y}_{\alpha c}) > 0 \quad (24)$$

then $V(\bar{y}_{\alpha c})$ is more efficient otherwise $V(\bar{y}_{RC})$ is more efficient.

Comparing (12) and (20)

$$V(\bar{y}_{\alpha c}) > V(\bar{y}_{\alpha s}) \Rightarrow V(\bar{y}_{\alpha c}) - V(\bar{y}_{\alpha s}) > 0 \quad (25)$$

then $V(\bar{y}_{\alpha s})$ is more efficient otherwise $V(\bar{y}_{\alpha c})$ is more efficient

Other criteria for detecting the more efficient estimator include; the Standard Error (SE), coefficient of Variation (CV) and Percentage Relative Efficiency (PRE)

Standard Error (SE)

$$SE(\hat{\bar{y}}) = \sqrt{MSE(\hat{\bar{y}})} \quad (26)$$

Coefficient of Variation (SE)

$$CV(\hat{\bar{y}}) = \frac{SE(\hat{\bar{y}})}{\bar{Y}} \times 100 \quad (27)$$

The estimator with the lowest CV (%) and SE are adjudged to be the most efficient estimator

Relative Efficiency (RE)

The relative efficiency of the separate modified estimator with respect to the combined modified estimator is

$$PRE = \frac{V(\bar{y}_{\alpha c})}{V(\bar{y}_{\alpha s})} \times 100 \quad (28)$$

$(\hat{\bar{y}}_{\alpha c})$ will be more efficient than $(\hat{\bar{y}}_{\alpha s})$ if and only if $V(\hat{\bar{y}}_{\alpha c}) < V(\hat{\bar{y}}_{\alpha s})$ or PRE < 100%

EMPERICAL COMPARISON

To examine the performance of the modified ratio estimators with the conventional ratio estimators, we have considered two populations. (Population I: Y= Household Expenditure, X= Household income. Population II: Y= Students CGPA, X= Students reading hours). The statistical description of the population is given below.

Table 1: Summary of Information of the study Population and Samples (Population I)

	N	N	Stratum I	Stratum II	Stratum III
			n_1=73	n_2=20	n_3=4
			N_1=183	N_2=50	N_3=11
			$\bar{X}_1$=51656	$\bar{X}_2$=123220	$\bar{X}_3$=249091
			$\bar{Y}_1$=50197	$\bar{Y}_2$=99700	$\bar{Y}_3$=158182
			θ_1=0.008	θ_2=0.03	θ_3=0.158
			S_{1y}^2=501906393	S_{2y}^2=231250000	S_{3y}^2=4691666667
Population I	244	97	S_{1x}^2=2705688356	S_{2x}^2=657556441	S_{3x}^2=2158333333
			S_{1y}=22403.267	S_{2y}=15206.906	S_{3y}=68495.741
			S_{1x}=52016.231	S_{2x}=25642.863	S_{3x}=46457.866
			S_{1xy}=465015221	S_{2xy}=207944079	S_{3xy}=2308333333
			$\bar{x}_1$=58247	$\bar{x}_2$=122823	$\bar{x}_3$=242500
			$\bar{y}_1$=49384	$\bar{y}_2$=101250	$\bar{y}_3$=177500
			ρ_1=0.399	ρ_2=0.533	ρ_3=0.725
			W_1=0.75	W_2=0.205	W_3=0.045
			R_1=0.9718	R_2=0.8091	R_3=0.6350

Table 2: Estimate of Population Characteristics under Stratified Random Sampling (Population I)

Estimator	*Mean*	*Bias*	*Variance*	*SE*	*CV*(%)	*PRE*(%)	Rank
$\hat{\bar{y}}_{RS}$	61882.93	240.6192	10184253.6	3191.277	5.15	33.255	4th
$\hat{\bar{y}}_{RC}$	62017.34	109.015	8261488.98	2874.281	4.63	39.650	3rd
$\hat{\bar{y}}_{\alpha s}$	64885.78	21.07371	3386821.1	1840.330	2.83	300.70	2nd
$\hat{\bar{y}}_{\alpha c}$	64934.44	11.10107	3275689.68	1809.886	2.78	252.20	1st

Table 3: Estimates of Population Characteristics for different Values of α (Population I).

		$\alpha = 0$	$\alpha_{opt} = 0.22$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$
	Mean	65789.37	64885.78	64764.55	63772.51	62812.277	61882.
	Bias	0	21.07371	25.33999	73.88986	145.6496	240.61
$\hat{\bar{y}}_{\alpha s}$	Variance	4051287.1	3386821.1	3598630.74	40347906.37	6341113.98	10184
	SE	2012.7809	1840.330	1897.005	2009.454	2518.156	3191.2
	CV (%)	3.06***	2.83*	2.93**	3.15	4.01	5.15
	Mean	65781.75	64934.44	64819.75	63871.82	62937.75	62017.
	Bias	0	11.10107	13.1778907	35.7396824	67.6853753	109.01
$\hat{\bar{y}}_{\alpha c}$	Variance	4051287.1	3275689.68	3357943.96	3828529.90	5463044.908	82614
	SE	2012.7809	1809.886	1832.46936	1956.663	2337.316	2874.2
	CV (%)	3.06***	2.78*	2.83**	3.07	3.71	4.63

Note: * implies the most efficient estimator, ** indicates second competing estimator and *** indicates the third efficient estimator among the class of estimators. α_{opt} Indicates the optimal value of α that maximizes efficiency.

Table 4: Summary of Information of the Study Population and Samples (Population II)

	N	N	Stratum I	Stratum II	Stratum III	Stratum IV
			n_1=94	n_2=49	n_3=43	n_4=25
			N_1=200	N_2=105	N_3=92	N_4=53
			$\bar{X}_1$=6.45	$\bar{X}_2$=6.25	$\bar{X}_3$=6.02	$\bar{X}_4$=5.66
			$\bar{Y}_1$=2.95	$\bar{Y}_2$=2.94	$\bar{Y}_3$=2.89	$\bar{Y}_4$=2.88
			θ_1=0.0056	θ_2=0.0108	θ_3=0.0123	θ_4=0.0211
			S^2_{1y}=0.678618	S^2_{2y}=0.641414	S^2_{3y}=0.871047	S^2_{4y}=0.838
Population II	450	211	S^2_{1x}=13.9439	S^2_{2x}=11.6234	S^2_{3x}=12.4253	S^2_{4x}=18.69
			S_{1y}=0.8237827	S_{2y}=0.8008832	S_{3y}=0.9332989	S_{4y}=0.915
			S_{1x}=3.734153	S_{2x}=3.409310	S_{3x}=3.524953	S_{4x}=4.323
			S_{1xy}=2.2879	S_{2xy}=1.4132	S_{3xy}=2.61879	S_{4xy}=3.623
			$\bar{x}_1$=6.32	$\bar{x}_2$=6.95	$\bar{x}_3$=5.83	$\bar{x}_4$=6.24
			$\bar{y}_1$=3.13	$\bar{y}_2$=3.07	$\bar{y}_3$=2.94	$\bar{y}_4$=3.03
			ρ_1=0.646	ρ_2=0.857	ρ_3=0.724	ρ_4=0.882
			W_1=0.444	W_2=0.233	W_3=0.204	W_4=0.117
			R_1=0.4573	R_2=0.4704	R_3=0.48	R_4=0.5088

Table 5: Estimate of Population Characteristics under Stratified Random Sampling (Population II)

Estimator	*Mean*	*Bias*	*Variance*	*SE*	*CV*(%)	*PRE*(%)	Rank
$\hat{\bar{y}}_{RS}$	3.0024	0.007170027	0.003924041	0.062642166	2.086	26.309	4th
$\hat{\bar{y}}_{RC}$	2.8286	0.001674389	0.003095000	0.055632724	1.966	29.906	3rd
$\hat{\bar{y}}_{\alpha s}$	3.0445	0.001570027	0.001032404	0.052680639	1.055	380.08	2nd
$\hat{\bar{y}}_{\alpha c}$	3.0354	0.000151200	0.00092562	0.03042400	1.002	334.37	1st

Table 6: Estimates of Population Characteristics for different Values of α (Population II)

		$\alpha = 0$	$\alpha_{opt} = 0.1$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	
$\bar{\bar{y}}_{\alpha s}$	Mean	3.0593	3.0445	3.0439	3.0294	3.0155	3.0024
	Bias	0	0.0015700	0.00173377	0.0021733	0.004318	0.007170027
	Variance	0.00179751	0.0010324	0.00152024	0.0019922	0.001999	0.003924041
	SE	0.0423970	0.0526806	0.03899033	0.0446345	0.044715	0.06264216
	CV (%)	1.385***	1.055*	1.280**	1.473	1.483	2.086
$\bar{\bar{y}}_{\alpha c}$	Mean	3.0593	3.0354	2.9999	2.9417	2.8846	2.8286
	Bias	0	0.0001512	0.00015649	0.0004671	0.000931	0.00155073
	Variance	0.0017975	0.0009256	0.00093893	0.0009522	0.001637	0.00309500
	SE	0.0423970	0.0304240	0.03064196	0.0308591	0.042867	0.05563272
	CV (%)	1.385***	1.002*	1.021**	1.049	1.403	1.966

Note: * implies the most efficient estimator, ** indicates second competing estimator and *** indicates the third efficient estimator among the class of estimators. α_{opt} Indicates the optimal value of α that maximizes efficiency.

Discussion Of Results

In this study, we applied an estimator proposed by Srivastava (1967) by stratifying the populations and applying SRSWOR scheme. The Biases and Variances of the proposed estimators were derived using first order approximation procedure and numerical results of the properties were obtained for efficiency comparison to the conventional separate ratio-type estimator and combined ratio-type estimator using data generated from two populations as presented in Tables 1 and 4. The assessments of the efficiency gains were done using SEs, CVs and PREs. From all the numerical results obtained in Tables 2 and 5, the proposed estimators have smallest Variances, SEs, CVs, and largest PREs as compared to other estimators in the study.

Table 2 and 5 presents the results of the efficiency and precision comparisons among the proposed and the conventional ratio estimators using the data obtained from population 1 and 2. The results revealed that the proposed estimators have minimum variances, SEs, CVs and larger PREs compared to the conventional estimators considered, and it is quite evident that there is a substantial gain in efficiency by using the proposed separate $\bar{y}_{\alpha s}$ and combined $\bar{\bar{y}}_{\alpha c}$ estimators over the traditional estimators $\hat{\bar{y}}, \hat{\bar{y}}_{RS}, \hat{\bar{y}}_{RC}$.

Table 3 and 6 provides the wide ranges of α and its optimum values for which the proposed class of estimators and are more efficient than the traditional estimators. It is also observed from Table 3 and 6 that there is a scoop for choosing the values of to obtain better estimators than. This shows that even if the scalar deviates from its optimum values, the suggested estimators will yield better estimates than. It is therefore worthy to conclude that when =0, the estimator tends to SRSWOR and when =1, it becomes a classical Ratio estimator. It is noticeable that the bias increases as and tends to zero as. However, the CV is minimum at in all cases on table 3 and 6. This suggests that the choice of should be determined by the data set as it may be optimized.

Conclusions

The ratio estimator is used to improve the performance of the estimator based on stratified random sampling, whenever there is an auxiliary variable which is positively correlated with that of the study variable and its parameters are known. We have applied the Srivastava (1967) estimator to stratified random sampling and we obtained the expressions for the bias and variances for both the separate and the combined estimator. By this estimators, the Bias and the variance of the modified estimators has been compared with that of the traditional separate and combined ratio estimate in theory and by this comparison it has been found that in all conditions the proposed estimators has a smaller variances, SEs, CVs and larger PREs than the traditional separate and combined ratio estimate. This theoretical result has also been satisfied by data set computations of two populations' parameters.

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