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An Empirical Analysis of the Random Walk and Stock Market Efficiency Hypotheses in the Nigerian Stock Market: Evidence from Hurst Exponent and ARGARCH Model

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Abstract

This empirical study examines the validity of the Random Walk Hypothesis (RWH) and the Efficient Market Hypothesis (EMH) within the Nigerian stock market, utilizing daily All Share Index (ASI) data from the Nigerian Stock Exchange spanning from January 2, 1998, to December 31, 2024. The analytical approach integrates the Augmented Dickey-Fuller (ADF) unit root test, the Random Walk Model, Hurst Exponent Rescaled Range (R/S) analysis, and the symmetric AR(1)-GARCH(1,1) model. Findings from the ADF test and random walk analysis reject the presence of a unit root and random walk behaviour in stock returns. Furthermore, both the yearly and full-sample Hurst exponent estimates, including the filtered R/S Hurst exponent, yield values greater than 0.5 (H>0.5), indicating that shocks to stock returns exhibit persistence and are not random. Similarly, the AR(1)-GARCH(1,1) model reveals that volatility shocks are highly persistent over time. Collectively, these consistent results across methodologies suggest that Nigerian stock returns are persistent, predictable, and deviate from randomness, implying that the market is weak-form inefficient. Consequently, the null hypotheses of a random walk and weak-form market efficiency are rejected. This inefficiency means that stock prices in Nigeria are not fully reflective of available information, making them predictable, prone to mispricing, and vulnerable to arbitrage, speculation, and manipulation. The study recommends strategies such as short-term trading for investors, timely dissemination of market information by operators, mitigation of trading frictions, and the development of policies to promote active and vibrant stock market activities.

Keywords: Hurst Exponent, Hypothesis, Market Efficiency, Random Walk, Unit Root, Stock Prices, Nigeria.

1. INTRODUCTION

The stock market serves as a platform for the buying and selling of existing securities, which represent the values of shares and bonds issued by firms and government institutions. The efficiency of the stock market is demonstrated when the prices of these securities, the trading mechanisms, and the allocation of ownership accurately reflect all relevant information. This ensures that transactions occur at fair market values, with minimal transaction costs, and that ownership is appropriately transferred (Udoka, 2012). The stock market plays a vital role in the financial system by mobilizing savings from both the public and private sectors and channeling these resources into productive investment ventures. It also safeguards investments from fraudulent and corrupt practices, facilitates transactions in government securities, promotes transparency between investors and industries, and acts as an information conduit for all market participants (Olabisi et al., 2017).

In additional, the stock market supports capital formation by efficiently allocating resources and reducing the risks associated with illiquidity, thereby enhancing investment opportunities. It provides a structured platform for trading existing securities and issuing new ones, fulfilling its fundamental function of resource mobilization and allocation (Abina, 2019). For the stock market to perform these roles effectively, it must operate efficiently, that is, security prices must fully reflect all available information (Afrafayia, 2018; Aliyev, 2019).

Stock market efficiency refers to a market condition in which the prices of stocks are determined solely by the forces of demand and supply, based on the information available at any given time. This efficiency is shaped by the flow of market information and the competitive behavior of investors and traders, whose decisions and investment patterns influence overall market effectiveness (Pervez et al., 2018). In an efficient market, all participants have equal access to relevant information, making it impossible for any trader or investor to consistently outperform the market or make unfair gains. Price movements in such a market are unpredictable, as new information is quickly and uniformly reflected in stock prices (Kuhe & Akor, 2021).

The Efficient Market Hypothesis (EMH), developed by Fama (1970), supports this view by asserting that in an efficient market, expected profits cannot be systematically earned through any specific trading strategy since all available information is already priced in. Closely related to this is the Random Walk Hypothesis (RWH), which posits that stock prices move in a random and unpredictable manner. According to RWH, successive changes in stock prices are independent and identically distributed, implying that past prices provide no useful information for predicting future prices, and that price movements do not follow any meaningful trend (Fama, 1970). Based on the nature and extent of information reflected in prices, Fama (1970) classified the EMH into three forms: the weak form, which considers only historical price information; the semi-strong form, which incorporates all publicly available information including financial reports and public announcements; and the strong form, which includes all information public, private, and even undisclosed. In a complementary framework, Adelegan (2003) categorized market efficiency into three types: operational, allocative, and pricing efficiency. Operational efficiency refers to the ability of the market to execute transactions quickly, accurately, and at minimal cost. Allocative efficiency ensures that financial resources are directed to their most productive uses, with investment opportunities going to the highest bidders, thereby maximizing the economy's use of scarce resources. Pricing efficiency, on the other hand, implies that security prices reflect all available information instantly and without bias, such that investors can only expect to earn returns that are proportionate to the risk undertaken.

The Random Walk Hypothesis (RWH) posits that stock prices move in a random and unpredictable manner, without following any discernible pattern. As such, relying on recent trends in stock prices to guide investment decisions is ineffective (Adelegan, 2003). The observation that stock prices have risen over the past few days does not provide any meaningful insight into today's or tomorrow’s price movements. Consequently, investors who attempt to forecast future prices or determine optimal times to buy or sell based solely on past price trends are likely engaging in a futile effort. This is because security prices do not exhibit repetitive patterns or regularities over time that could be exploited for predictive purposes. Instead, each price change occurs independently of the last, and the behavior of stock prices reflects a random process. When a market's stock prices exhibit such randomness, it is considered weak-form efficient. Conversely, if prices can be predicted based on historical trends, the market is deemed weak-form inefficient (Kuhe & Akor, 2021).

While extensive research has been conducted on the Efficient Market Hypothesis (EMH) and the Random Walk Hypothesis, most of these studies have concentrated on well-developed and highly organized stock markets, often overlooking emerging and developing markets such as those in Africa. It is commonly assumed that in developing markets, stock prices tend not to follow a random walk, and the markets exhibit weak-form inefficiency. However, this assumption requires further empirical validation. There is a critical need to reassess, re-investigate, and document the efficiency of African stock markets to provide a clearer understanding of their dynamics.

Against this backdrop, the present study aims to empirically examine the Efficient Market Hypothesis and the Random Walk Hypothesis in the context of the Nigerian Stock Market (NSM), using daily closing values of the All Share Index (ASI) of the Nigerian Stock Exchange (NSE). Specifically, the study seeks to: (i) investigate the validity of the Efficient Market Hypothesis in the Nigerian stock market; and (ii) assess whether Nigerian stock returns follow a random walk. The structure of the paper is as follows: Section 2 reviews relevant empirical literature; Section 3 outlines the materials and methods employed;

Section 4 presents and discusses the results; and Section 5 provides the concluding remarks.

2. LITERATURE REVIEW

A number of empirical studies have examined the validity of the Efficient Market Hypothesis (EMH) and the Random Walk Hypothesis (RWH) across various stock markets, yielding mixed results. In Nigeria, Nwosa and Oseni (2011) tested the weak-form efficiency using data from 1986 to 2010 through serial autocorrelation and regression analysis. Their findings indicated that stock prices did not follow a random walk, suggesting that the market was informationally inefficient. In contrast, Ajao and Osayuwu (2012), using data from 2001 to 2010 and employing serial correlation, Ljung-Box, and Box-Pierce Q-statistics, found evidence of randomness and independence in stock price changes, thereby supporting weak-form efficiency in the Nigerian capital market.

Beyond Nigeria, Owido et al. (2013) used both non-parametric tests and a GARCH model to assess the Nairobi Securities Exchange and concluded that daily stock returns were not random, indicating weak-form inefficiency. McKerrow (2013) examined five frontier African markets and found mixed evidence: Botswana, Ghana, and Mauritius supported the random walk hypothesis, while Namibia and Ivory Coast did not. Similarly, Alkhatibu and Harsheh (2013) tested the Palestine Exchange (PEX) using serial correlation, unit root, and runs tests, concluding that price movements were not random, and the market was weak-form inefficient.

Maximillian and Gwahula (2018) investigated the Efficient Market Hypothesis (EMH) in the Dar es Salaam Stock Exchange (DSE) using daily closing stock prices from January 2009 to March 2015. Utilizing a combination of statistical tools including serial correlation, Ljung-Box Q-statistics, unit root test, runs test, and the variance ratio test, they found consistent evidence across all methods that daily stock returns did not behave randomly. Consequently, the study concluded that the DSE was not weak-form efficient. Similarly, Kuhe and Akor (2021) examined the random walk hypothesis in the Nigerian Stock Market using daily stock returns from January 1998 to December 2019. The study employed the Augmented Dickey-Fuller (ADF) test, Ljung-Box Q-statistics, runs test, and a robust variance ratio test. Their findings were uniform across all tests, indicating that stock returns were non-random. The Nigerian market was thus found to be weak-form inefficient, with stock prices deemed predictable, mispriced, and vulnerable to arbitrage, speculation, and manipulation.

In contrast, Gbalam and Nelson (2019) assessed the weak-form efficiency of the Nigerian Stock Exchange using daily, weekly, and monthly All Share Index data. Employing unit root testing, random walk analysis, Jarque-Bera test for normality, and graphical tools, their results suggested that the Nigerian stock market exhibited random walk characteristics and was weak-form efficient. However, their findings have been critiqued for methodological limitations, particularly the use of stock price levels rather than stock returns. Since stock market efficiency is more accurately assessed using returns, the reliability of their conclusion is questionable.

Gbalam and Nelson (2019) examined the Nigerian Stock Exchange using daily, weekly, and monthly All Share Index data, applying unit root and random walk tests. Their results supported weak-form efficiency; however, their study was criticized for relying on stock prices rather than returns, which undermines the robustness of their conclusions. More technically robust approaches were adopted by Olunkwa et al. (2023), who employed ARMA-EGARCH models with Student-t distributions to estimate volatility dynamics in daily stock returns. Their results highlighted strong volatility persistence and indicated that Nigerian stock prices are predictable to some degree. Other studies further confirm this trend. Bello (2020) applied mean-reverting GARCH models to Nigerian stock data from 2008 to 2018 and found evidence of volatility persistence, suggesting market inefficiency. Onwukwe et al. (2009) also observed volatility clustering and leverage effects using GJR-GARCH models, identifying a significant dependency in return series. Similarly, Usman et al. (2017) compared eleven GARCH models using monthly data from 1996 to 2015. Their findings revealed varying performance across models depending on training and testing phases, with significant implications for volatility predictability.

Furthermore, Emenogu et al. (2020) modeled the stock returns of Total Nigeria Plc and GTBank using ARMA-GARCH models and found strong evidence of autocorrelation and volatility persistence, further affirming weak-form inefficiency. Shittu et al. (2019) employed a BEKK-MGARCH model to examine volatility transmission between Nigerian stock returns and macroeconomic variables such as exchange rates and inflation. Their study revealed strong co-volatility and feedback effects, suggesting that external economic variables significantly influence market behaviour. Additional support for weak-form inefficiency is found in Nageri and Abdulkadir (2019), who compared pre- and post-financial crisis stock market behaviour in Nigeria. Their findings showed persistent inefficiency before and after the 2007-2009 global financial crisis, although a degree of semi-strong efficiency was observed during the crisis period. Collectively, these studies indicate that despite occasional evidence of efficiency, the Nigerian stock market generally fails to meet the criteria for weak-form efficiency due to predictable return patterns and persistent volatility. This highlights the need for continuous reassessment using robust models such as AR-GARCH and tools like the Hurst exponent to understand evolving market dynamics.

A thorough review of the empirical literature reveals a lack of consensus among researchers concerning the behaviour of stock market returns. Findings from both developed and developing economies, including Nigeria, have been mixed and often contradictory. While some studies support the existence of weak-form efficiency and fail to reject the random walk hypothesis particularly in emerging markets, others present contrary evidence. This study contributes to the ongoing discourse by revisiting and critically evaluating the Efficient Market Hypothesis and the Random Walk Hypothesis in the context of the Nigerian stock market. It extends the existing body of knowledge by employing more robust statistical techniques and utilizing more recent and comprehensive data.

3. MATERIALS AND METHODS

3.1 Data Source

The data used in this study are the daily closing all share prices (ASI) of the Nigerian Stock Exchange (NSE) obtained from www.nse.org.ng and spanned from 2nd January, 1998 to 31st December, 2024. The daily stock market prices are converted to daily log returns using the following formula:

$$r_t = 100.\ln \Delta R_t \quad (1)$$

where r_t is the daily stock return series, $\Delta$is the first difference operator $R_t$is the daily closing market index at the current day t.

3.2 Methods of Data Analysis

The following statistical tools are employed in the analysis of data in this work.

3.2.1 Descriptive statistics and normality measures

The mean of the daily stock return is computed as:

$$\bar{r} = \frac{1}{n}\sum_{i=1}^{n} r_i \quad (2)$$

The sample standard deviation of returns over a given period of time is computed using the following formula:

$$\hat{\sigma} = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(r_t - \bar{r})^2} \quad (3)$$

where $\bar{r}$ is the mean return, n is the sample size.

Jarque-Bera test is a normality test of whether a given sample data have the skewness and kurtosis similar to that of a normal distribution. The test was proposed by Jarque and Bera (1980, 1987) and test the null hypothesis that the series is normally distributed. Given a return series $\{r_t\}$ the test statistic JB is defined as:

$$JB = \frac{T}{6} ¿$$

where g_1 is the sample skewness given as:

$$g_1 = \frac{\mu_3}{\mu_2^{3/2}} = ¿$$

and g_2 is the sample kurtosis defined by:

$$g_2 = \frac{\mu_4}{\mu_2^2} = ¿$$

$H_0: \hat{\mu}_3 = 0$ and $\hat{\mu}_4 = 0$ (i.e., r_t follows a normal distribution)

$H_1: \hat{\mu}_3 \neq 0$ and $\hat{\mu}_4 \neq 0$ (i.e., r_t does not follows a normal distribution).

The test rejects the null hypothesis if the p-value of the JB test statistic is less than $\alpha = 0.05$ level of significance.

3.2.2 Augmented Dickey-Fuller (ADF) unit root test

This study employs the popular parametric Augmented Dickey-Fuller (ADF) unit root test due to Dickey and Fuller (1979). The ADF test constructs a parametric correction for higher-order correlation by assuming that the stock return series (r_t) follows an AR(p) process and adding lagged difference terms of the dependent variable to the right-hand side of the test regression:

$$\Delta r_t = \alpha r_{t-1} + x_t' \delta + \phi_1 \Delta r_{t-1} + \phi_2 \Delta r_{t-2} + \ldots + \phi_p \Delta r_{t-p} + \varepsilon_t \quad (7)$$

where x_t are optional exogenous regressors which may consist of constant, or a constant and trend, α and δ are parameters to be estimated, and the ε_t are assumed to be white noise. The null and alternative hypotheses are written as:

$H_0: \alpha = 0$ (i.e., the series contains a unit root)

$H_1: \alpha < 0$ (i.e., the series is stationary)

and evaluated using the conventional $t-¿$statistic for α:

$$t_\alpha = \hat{\alpha} / SE(\hat{\alpha}) \quad (8)$$

where $\hat{\alpha}$ is the estimate of α, and $SE(\hat{\alpha})$ is the coefficient standard error. The test rejects the null hypothesis of a unit root if the ADF test statistic is smaller (i.e., more negative) than the asymptotic critical value at the designated test size or if the p-value of the ADF test statistic is less than $\alpha = 0.05$.

3.2.3 The random walk model

A random walk is a non-stationary process without a drift, with a drift or with a drift and deterministic time trend. Consider a simple random walk model without a drift below:

$$r_t = \alpha r_{t-1} + \varepsilon_t \quad (9)$$

where ε_t is independently identically distributed with mean zero and variance σ_ε^2. The OLS of α is computed as:

$$\hat{\alpha} = \left(\sum_{t=2}^{T} r_{t-1}^2 \right)^{-1} \left(\sum_{t=2}^{T} r_t r_{t-1} \right) \quad (10)$$

The model checks the following pairs of hypothesis

$H_0: \alpha = 0$ (i.e., r_t is a random walk)

$H_0: \alpha < 0$ (i.e., r_t is a stationary process)

Consider a random walk model with a drift which is an extension of equation (9)

$$r_t = \mu + \alpha r_{t-1} + \varepsilon_t (11)$$

If $\mu \neq 0$, (r_t) has a linear deterministic trend and μ is called a drift. The OLS of α in (3.11) is given by

$$\hat{\alpha} = ¿¿$$

$$where\ \bar{r}_{-1} = \frac{1}{T-1} \sum_{t=1}^{T} r_t \wedge \bar{r} = \frac{1}{T-1} \sum_{t=2}^{T} r_t$$

The model tests the following pairs of hypothesis

$H_0 : \alpha = 0, \mu \neq 0$ (i.e., r_t is a random walk around a drift)

$H_0 : \alpha < 0, \mu \neq 0$ (i.e., r_t is a level stationary process)

Consider a random walk model with a drift and linear trend below

$$r_t = \mu + \beta t + \alpha r_{t-1} + \varepsilon_t (13)$$

If $\mu = 0$ and $\alpha = 1$, (r_t) contains at most a constant trend and

$$T(\hat{\alpha} - 1) \underset{\rightarrow}{D} ¿¿$$

$$where\ w_1(r) = w(r) - \int_0^1 w(s)\, ds$$

The F-statistic to test

$$(\mu, \alpha) = (0, 1) \underset{\rightarrow}{D} \frac{1}{2} ¿$$

But if $\beta \neq 0$ and $\alpha = 1$, (r_t) contains a quadratic trend and the situation is analogous to the case where $\mu \neq 0$ and $\alpha = 1$. However, Dickey-Fuller test does not admit $\beta \neq 0$ and $\alpha = 1$. If $(\beta, \alpha) = (0, 1)$, then

$$T(\hat{\alpha} - 1) \underset{\rightarrow}{D} ¿¿$$

The F-statistic to test

$$(\mu, \alpha) = (0, 1) \underset{\rightarrow}{D} \frac{1}{2} ¿$$

The random walk model with drift and linear time trend tests the following pairs of hypothesis

$H_0 : \alpha = 0, \beta \neq 0$ (i.e., r_t is a random walk around a trend)

$H_0 : \alpha < 0, \beta \neq 0$ (i.e., r_t is a trend stationary process)

These statistics can be used in testing difference stationarity against trend stationarity in the presence of a deterministic trend. In testing the random walk hypothesis, if the calculated value of the ADF test statistic exceeds the test asymptotic critical values, the hypothesis that $\alpha=0$is rejected indicating that the series is stationary. If however, the calculated ADF test statistic does not exceed the critical value, the null hypothesis is not rejected and the series is said to be non-stationary.

3.2.4 The Hurst Exponent: Rescaled Range (R/S) Analysis

Hurst exponent (H) is a statistical tool used for classifying time series that provides a criterion for long-term memory (long range dependence) and is thus known as an index of dependence. Hurst exponent (H) is a suitable tool for distinguishing a non-random time series from a random time series regardless of its distribution (Hurst, 1951).

Let R_t denotes the daily closing share price of a stock at time t and let r_t be the logarithmic return series defined in Equation (1). The rescaled range (R/S) statistic refers to the range of partial sums of deviations of times series from its mean, rescaled by its standard deviation. Consider a sample of continuously compounded asset returns $\{r_1, r_2, r_3, \ldots, r_n\}$ and let $\bar{r}$ denote the sample mean defined in Equation (2) where n is the number of observations. Then the R/S statistic is defined as follows:

$$R_n=\frac{1}{s_n}\left[\max_{1\le t\le n}\sum_{t=1}^{n}(r_t-\bar{r})-\min_{1\le t\le n}\sum_{t=1}^{n}(r_t-\bar{r})\right](16)$$

where R_n is an adjusted range which is defined as the difference between the maximum and minimum value of the cumulated deviations, s_n is the standard deviation of returns given by

$$s_n=\left[\frac{1}{n-1}\sum_{t=1}^{n}(r_t-\bar{r})^2\right]^{1/2}(17)$$

The adjusted range R_n is standardized by the standard deviation s_n and forms a rescaled range as R_n/s_n. Hurst (1951) found that R/S for many records in time is well described by the following empirical relation:

¿

Peters (1994) used the following formula for the estimation of Hurst exponent.

$$E\left(\frac{R_n}{s_n}\right)=Cn^H(20)$$

where H is the Hurst exponent, n is the time span of the observations and C is a constant. The Hurst exponent H is the slope of the plot of each range log¿¿ versus each range $\log(n)$. Analysis of Hurst exponent estimation and its accuracy is given in (Resta, 2012).

The numerical values of Hurst exponent range from 0 to 1 based on which time series is classified into three categories (Peters, 1994).

1) When the value of Hurst exponent ranges from 0 to 0.5 ($0<H<0.5$), it indicates that the series is mean-reverting with anti-persistent

behaviour. The values of the series under this condition are negatively correlated for all periods and the return series will have a short-term memory indicating a weak-form efficient market.

2) When the value of Hurst exponent equals 0.5 ($H=0.5$), it indicates a true random walk process (a Brownian time series) and under this condition the time series values are not correlated.
3) When the value of Hurst exponent ranges from 0.5 to 1 ($0.5<H<1$), it indicates that the series is persistent. When this happens, the variance process of the time series is persistent with long-term memory (long range dependence). Under this condition, the time series values are positively correlated together. A long-term memory in the series indicates an inefficient market. If the value of H is closer to unity, it indicates a high risk of large and abrupt changes (Hurst, 1951).

3.2.5 The GARCH model Approach

The second procedure also employs the classical R/S analysis. However, the R/S analysis is now applied to the log-return time series filtered by an AR(1)-GARCH(1,1) process. The autoregressive process of order one AR (1) is expressed as:

$$r_t=\phi_0+\phi_1 r_{t-1}+\varepsilon_t (21)$$

where r_t is the log-return series at time t, ϕ_0 is a constant, ϕ_1 is an autoregressive parameter to be determined and ε_t is an error term.

The mean equation and the conditional variance equation of an AR(1)-GARCH (1,1) process are given respectively as:

$$r_t=\mu+\phi_1 r_{t-1}+\varepsilon_t (22)$$

$$h_t=\omega+\alpha_1\varepsilon_{t-1}^2+\beta_1 h_{t-1} (23)$$

with constraints $\omega>0,\alpha_1\geq 0,\beta_1\geq 0,\wedge\alpha_1+\beta_1<1$ to ensure conditional variance to be positive as well as stationary. Where h_t is the conditional variance, μ and ω are the constants for the mean and variance equations respectively, α_1 and β_1 are the ARCH and GARCH terms to be estimated.

After fitting the AR(1)-GARCH (1,1) model to the data, a serial correlation test is conducted on the residuals of the estimated model and if the p-values of the serial correlation test are found to be insignificant, then the estimated model is said to have filtered the data for short-range correlation. The R/S analysis is then applied to the standardized residuals $\varepsilon_t^{¿}$ to perform test for long-range dependence where $\varepsilon_t^{¿}$ is defined as

$$\varepsilon_t^{¿}=\frac{\varepsilon_t}{\sqrt{h_t}}(24)$$

4. RESULTS AND DISCUSSION

4.1 Summary Statistics and Normality Measures of Daily Stock Returns

To understand the descriptive and distributional characteristics of Nigerian daily stock returns, summary statistics such as the mean, range, standard deviation as well as normality measures such as skewness, kurtosis and Jarque-Bera statistic are employed and the results are presented in Table 1.

Table 1: Summary Statistics and Normality Measures of Daily Stock Returns

Mean	Range	SD	Skew.	Kurt.	JB	P-value	N
0.01794	10.34246	0.423689	0.012713	13.82553	32423.34	0.00000	6640

From the results of summary statistics and normality measures reported in Table 1, observe that the daily mean return is positive (0.028103) indicating gain in the stock market during the trading period under consideration. The high values of the daily range return and the standard deviation show a high level of variability of price changes in the Nigerian stock market.

The skewness of the stock market return series is negative indicating that the stock index returns are flatter and skewed to the left as compared to the normal distribution. This implies that the asymmetric tail of the return series extends more towards negative values than positive ones. The kurtosis value of the returns is very high indicating that the stock return distribution has sharp peak when compared to a normal distribution. This suggests that the return distribution is heavy tailed. Additionally, the high value of Jarque-Bera statistic with marginal probability value of 0.0000 confirms the deviation of the daily stock returns from normality.

4.2 Graphical Examination of the Series

The graphs presented in Figure 1 depict the time series behaviour of the Nigerian All Share Index (ASI) over the period of 1998 to 2024. The left panel shows the level series of the ASI, while the right panel illustrates the log returns. This visualization is used to assess market trends, volatility, and suitability of a series for econometric modeling, particularly in evaluating stationarity and market efficiency.

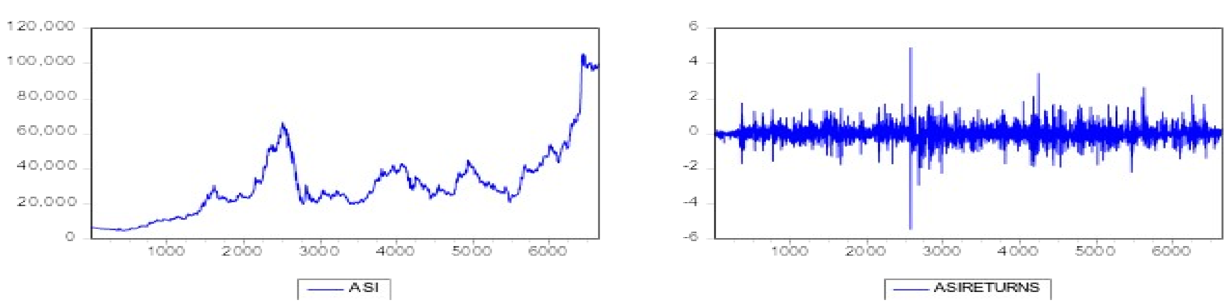

Figure 1: Time Plots of ASI and ASI Returns

From the time plots reported in Figure 1, the left panel displays the ASI level series, showing a clear long-term upward trend with periodic fluctuations marked by peaks and troughs, reflecting market cycles and episodes of boom and bust. The varying mean and variance indicate non-stationarity, a common trait in raw financial time series, necessitating transformation for robust econometric analysis. In contrast, the right panel presents the log returns of ASI, which exhibit fluctuations around a constant mean with relatively stable variance, apart from noticeable volatility spikes. This behaviour signifies stationarity and volatility clustering, making the transformed series appropriate for time series modeling using techniques such as GARCH.

4.3 Results of Unit Root Test for NSE Returns

Unit root test is a statistical test for investigating stationarity in time series. To investigate the random walk hypothesis (RWH), unit root test of stationarity (absence of randomness) is very necessary. In this work Augmented Dickey-Fuller (ADF) unit root test has been employed for this purpose. The result of the ADF unit root test is presented in Table 2.

Table 2: Results of ADF Unit Root Test for NSE Returns

Year	ADF Test Statistic	P-value	5% Critical value
1998	-14.6185	0.0000	-3.4280
1999	-10.1360	0.0000	-3.4280
2000	-8.8359	0.0000	-3.4280
2001	-8.9302	0.0000	-3.4281
2002	-9.9151	0.0000	-3.4281
2003	-8.1800	0.0000	-3.4280
2004	-8.7036	0.0000	-3.4281
2005	-9.2118	0.0000	-3.4276
2006	-9.1073	0.0000	-3.4282
2007	-9.6082	0.0000	-3.4286
2008	-7.7236	0.0000	-3.4381
2009	-8.3019	0.0000	-3.4282
2010	-12.1005	0.0000	-3.4280
2011	-11.7562	0.0000	-3.4286
2012	-14.0069	0.0000	-3.4283
2013	-13.2558	0.0000	-3.4282
2014	-9.2352	0.0000	-3.4289
2015	-9.6604	0.0000	-3.4282
2016	-9.0840	0.0000	-3.4285
2017	-11.3215	0.0000	-3.4285
2018	-13.7947	0.0000	-3.4281
2019	-12.1947	0.0000	-3.4282
2020	-14.7612	0.0000	-3.4284
2021	-17.2751	0.0000	-3.4283
2022	-15.6611	0.0000	-3.4282
2023	-14.2854	0.0000	-3.4285
2024	-22.2687	0.0000	-3.4284
Full Sample	-44.6514	0.0000	-3.4282

Table 2 presents the results of the Augmented Dickey-Fuller (ADF) unit root test conducted for both the annual data spanning 1998 to 2024 and the entire sample period. The ADF test statistics indicate that stock returns in the Nigerian stock market are stationary at the 1% level of significance. Consequently, the null hypothesis that the stock prices exhibit a unit root is rejected. This conclusion is supported by the ADF test statistics, which are more negative than the corresponding 5% critical values, as well as the statistically significant p-values associated with the test.

The Random Walk Hypothesis (RWH) is considered valid if stock market return series exhibit a unit root, indicating non-stationarity. However, while the presence of a unit root is a necessary condition for a random walk, it is not sufficient on its own. In other words, a series cannot follow a random walk if it is stationary, but the existence of a unit root alone does not conclusively confirm randomness. Given that the Augmented Dickey-Fuller (ADF) test results reject the null hypothesis of a unit root across all study periods, it implies that the Nigerian stock market return series are covariance-stationary. This lack of randomness suggests that the Nigerian Stock Exchange (NSE) does not exhibit weak-form efficiency, allowing investors to potentially predict future stock returns based on past information.

4.4 Hurst Exponent and GARCH Model Results

The investigation into the Random Walk Hypothesis (RWH) and the Efficient Market Hypothesis (EMH) in the Nigerian Stock Market (NSM) is further enriched through the application of Hurst exponent analysis and symmetric GARCH modeling. Table 3 presents the Hurst exponent variance for the entire sample period, providing insights into the long-term memory properties of the stock return series. Table 4 displays the conditional variance estimates derived from the symmetric AR(1)-GARCH(1,1) model, along with the Ljung-Box Q-statistics used to test for serial correlation in the model's residuals over the full sample period. Table 5 extends this analysis by presenting annual Hurst exponent variances alongside the conditional variances of Nigerian stock returns estimated using the GARCH(1,1) model. It also includes yearly estimates of shock persistence and the filtered Hurst exponent values obtained from the residuals of the AR(1)-GARCH(1,1) model, offering a comprehensive view of both the return dynamics and memory behaviour across time.

Table 3: Parameter Estimate of the Hurst Exponent (Full Sample)

Size	RS(avg)	log(Size)	log(RS)
5426	204.45	12.406	7.6756
2713	116.79	11.406	6.8678
1356	77.017	10.405	6.2671
678	55.628	9.4051	5.7977
339	32.203	8.4051	5.0091

169	21.266	7.4009	4.4105
84	14.204	6.3923	3.8282
42	9.0255	5.3923	3.1740
21	5.6677	4.3923	2.5028
10	3.3234	3.3219	1.7327

Regression Results (n = 10)

	Coefficient	Std. Error
Intercept	-0.31587	0.067761
Slope	0.63887	0.0080623

Estimated Hurst exponent = 0.638874

Note: Logs are to base two

As reported in Table 3, the estimated Hurst exponent for the full sample period is 0.638874. Additionally, the annual Hurst exponent values presented in Column 7 of Table 5 indicate that all estimated values exceed 0.5 ($H > 0.5$) across the individual years examined. This consistently high Hurst exponent suggests that the Nigerian stock return series exhibits persistence and long-term memory, or long-range dependence. The implication of $H > 0.5$ across all time periods is that the stock prices do not follow a random walk, indicating that the Nigerian stock market is not weak-form efficient. This lack of efficiency implies that the market is informationally inefficient, allowing arbitrage opportunities to persist and enabling informed or strategic investors to potentially outperform the market. These findings are consistent with previous studies such as Sensoy (2013), who documented long-range dependence in MENA markets including the Saudi Stock Market; Al-Shboul and Alsharari (2019), who identified similar characteristics in UAE stock markets; and Lamouchi (2020), who also found evidence of long memory in the return series of the Saudi Stock Market.

Table 4: Parameter Estimate of AR(1)-GARCH (1,1) Model (Full Sample)

Variable	Coefficient	Std. Error	z-Statistic	P-value
μ	-0.0150	0.0079	-1.8987	0.0598
ϕ_1	0.2837	0.0179	15.8492	0.0000
ω	0.0274	0.0039	7.0256	0.0000
α_1	0.3266	0.0264	12.3750	0.0000
β_1	0.6732	0.0148	45.4865	0.0000
v	4.6375	0.2794	16.5981	0.0000
$\alpha_1+\beta_1$	**0.9998**			

Ljung-Box Q-statistic Serial correlation Test Result

Lag	Q-statistic	P-value	Lag	Q-statistic	P-value
1	0.0085	0.926	11	8.5844	0.660
2	1.2277	0.541	12	8.6801	0.730
3	1.6386	0.651	13	9.3728	0.744
4	3.8668	0.424	14	9.5649	0.793
5	4.9086	0.427	15	9.6403	0.842
6	6.8584	0.334	20	11.854	0.921
7	8.0199	0.331	25	12.510	0.982
8	8.2687	0.408	20	13.770	0.995
9	8.2917	0.505	35	15.647	0.998
10	8.4283	0.587	36	15.727	0.999

The upper panel of Table 4 presents the parameter estimates from the symmetric AR(1)-GARCH(1,1) model, applied to the stock return series over the full sample period, incorporating non-Gaussian error distributions. All estimated parameters in the variance equation are statistically significant and adhere to the non-negativity conditions required for model validity. The positive and significant coefficients of the ARCH term (α_1) and the GARCH term (β_1) suggest that past volatility shocks carry explanatory power for current volatility, thereby confirming the presence of volatility clustering in the Nigerian stock returns. Furthermore, the sum of α_1 and β_1 is less than one (i.e., $\alpha_1+\beta_1<1$), indicating that the conditional variance process is stationary, stable, and mean-reverting, with persistent but ultimately diminishing volatility shocks.

The lower panel of Table 4 reports the Ljung-Box Q-statistics used to test for residual serial correlation in the fitted AR(1)-GARCH(1,1) model. The insignificance of the Q-statistic p-values ($p > 0.05$) confirms that the model has adequately filtered short-range dependence from the return series. To further assess long-range dependence, the Rescaled Range (R/S) analysis was applied to the standardized residuals, and the results are reported in the last row of Table 5.

Columns 1 to 6 of Table 5 display annual parameter estimates from the symmetric AR(1)-GARCH(1,1) models for each year and for the full sample period. Across all periods, the sum of the ARCH and GARCH coefficients remains below unity, reaffirming that the variance processes are stable, predictable, and stationary. However, given that these sums are close to one in most years, the findings also suggest a high degree of volatility persistence in the Nigerian stock market over time.

Considering the collective findings from the ADF unit root tests, Hurst exponent estimates, AR(1)-GARCH(1,1) model outputs, and the filtered Hurst exponent results (Tables 2 through 5), it is evident that Nigerian stock returns exhibit strong persistence, non-random behaviour, and predictability. These results point to weak-form inefficiency in the Nigerian stock market. Consequently, the null hypotheses of a random walk and weak-form efficiency are rejected for the period under review.

This observed inefficiency in the Nigerian Stock Exchange (NSE) implies that stock prices do not instantaneously reflect all historical information. As a result, investors may exploit available information to outperform the market and earn abnormal profits. This environment makes technical analysis a potentially valuable tool for traders and investors, as price patterns and historical data may contain exploitable signals for forecasting future market movements.

Table 5: Annual Parameter Estimates of GARCH (1,1) Models and Hurst Exponents

Year	μ	ω	α_1	β_1	$\alpha_1+\beta_1$	Hurst Exponent	Remark
1998	-0.0346	0.0018	0.0224	0.9337	0.9561	0.6917	Persistent
1999	-0.0425	0.0075	0.1858	0.8070	0.9928	0.7299	Persistent
2000	0.1360	0.0434	0.2806	0.6509	0.9315	0.7154	Persistent
2001	0.0582	0.1170	0.3811	0.4470	0.8281	0.5501	Persistent
2002	-0.0165	0.1143	0.2202	0.5779	0.7981	0.6344	Persistent
2003	0.1804	0.1144	0.4567	0.4372	0.8939	0.6220	Persistent
2004	0.0226	0.3652	0.1169	0.6431	0.7600	0.6726	Persistent
2005	-0.0221	0.0624	0.2626	0.6069	0.8695	0.6663	Persistent
2006	-0.0002	0.0937	0.4120	0.4003	0.8123	0.7382	Persistent
2007	0.0908	0.1359	0.3902	0.4347	0.8249	0.6638	Persistent
2008	-0.3877	0.1175	0.2711	0.5043	0.7754	0.5477	Persistent
2009	-0.1351	0.1626	0.4873	0.5113	0.9986	0.6610	Persistent
2010	0.0457	0.4793	0.1936	0.4361	0.6297	0.6559	Persistent
2011	-0.1038	0.0595	0.1444	0.7472	0.8916	0.6525	Persistent
2012	0.1129	0.0741	0.0238	0.8096	0.8334	0.6219	Persistent
2013	0.1005	0.0140	0.0706	0.9116	0.9822	0.6792	Persistent
2014	-0.0493	0.6719	0.3277	0.6688	0.9965	0.7218	Persistent
2015	-0.1678	0.2191	0.4535	0.5099	0.9634	0.5745	Persistent
2016	-0.0243	0.0829	0.1032	0.8162	0.9194	0.6921	Persistent
2017	0.0792	0.2146	0.3731	0.5129	0.8860	0.7133	Persistent
2018	-0.1200	0.2459	0.3634	0.5009	0.8643	0.6204	Persistent
2019	-0.1278	0.0888	0.3540	0.5988	0.9528	0.5742	Persistent
2020	-0.2541	0.1226	0.3226	0.6435	0.9661	0.6185	Persistent
2021	-0.3184	0.4274	0.2864	0.6788	0.9652	0.5674	Persistent
2022	-0.2752	0.3218	0.1897	0.7652	0.9549	0.7413	Persistent
2023	-0.1074	0.2247	0.4726	0.5272	0.9998	0.6688	Persistent
2024	-0.3641	0.5471	0.3678	0.6128	0.9806	0.6842	Persistent
Full	-0.2162	0.2462	0.3075	0.6923	0.9998	0.6389	Persistent

Sample		
Filtered R/S Hurst Exponent (Residuals)	0.6389	Persistent

5. CONCLUDING REMARKS

This study empirically investigated the Random Walk Hypothesis (RWH) and the Efficient Market Hypothesis (EMH) within the context of the Nigerian stock market, using daily stock return data from the Nigerian Stock Exchange spanning January 2, 1998, to December 31, 2024. The analysis employed the Augmented Dickey-Fuller (ADF) unit root test, the Hurst exponent, the Random Walk Model, and the symmetric AR(1)-GARCH(1,1) model as methodological tools. Results from the ADF test rejected the null hypothesis of a unit root, indicating that Nigerian stock returns are stationary and do not follow a random walk. Similarly, the Hurst exponent values for both the full sample and individual years consistently exceeded 0.5 ($H > 0.5$), indicating long-range dependence and return persistence. The filtered rescaled range (R/S) Hurst exponent also confirmed this result.

Furthermore, the AR(1)-GARCH(1,1) model estimation showed strong evidence of persistent volatility shocks in both the annual and full-period analyses, highlighting the existence of volatility clustering and mean-reverting behavior in stock return dynamics. Collectively, the findings from the ADF test, Hurst exponent analysis, and the AR(1)-GARCH(1,1) model provide consistent evidence that the Nigerian stock market is weak-form inefficient. The results indicate that stock returns are non-random, predictable, and potentially exploitable, leading to the rejection of the null hypotheses of random walk behavior and weak-form efficiency during the period under review.

The combined outcomes of these three methodological approaches strongly suggest that daily stock return series in the Nigerian stock market do not evolve randomly. Instead, they exhibit patterns that deviate from weak-form market efficiency. Based on the study's findings, it is concluded that the Nigerian stock market does not conform to the weak-form EMH. This inefficiency implies that stock prices in the Nigerian market do not fully and instantly incorporate historical information, leaving room for arbitrage opportunities and allowing informed investors to consistently outperform the market. The market is thus characterized by predictable price movements, consistent mispricing, susceptibility to speculation, and vulnerability to manipulation.

In light of these empirical findings, the following recommendations are proposed:

1. Stock market regulators should enforce timely and transparent disclosure of relevant information on the performance of securities to all market participants. This will help reduce information asymmetry and improve market integrity.

2. The government and Nigerian Stock Exchange (NSE) authorities should implement policies aimed at reducing

trading frictions, enhancing market liquidity, and promoting active participation in the stock market.

3. Given the weak-form inefficiency of the market, investors and traders are advised to focus on short-term investment strategies and technical analysis until market efficiency improves and price predictability declines.

4. Future research should focus on individual stocks listed on the Nigerian Stock Exchange to assess the efficiency levels at the firm level. This will provide deeper understanding of the application of trading strategies and help identify which securities are more prone to inefficiencies.

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